Characterization of Satellite Swarms Under Non-Keplerian Dynamics – Model Development

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Current Status

- Completion of new literature search for **non-Keplerian disturbance terms** and the applications in which they appear.
  - Particularly interest in applications involving **formation-flying satellites** (e.g., Geodesy).
- Further development of the framework to implement **new disturbance terms as loadable data files**.
  - Sources pulled directly from **NASA-JPL's NAIF and PO.DAAC databases** to obtain Earth orientation and geopotential data.
- Introduced additional disturbance terms to bring the dynamics in line with the International Earth Rotation and Reference System Service (**IERS**) **2010 Conventions**.

Review of Prior Discussion

The case of Keplerian dynamics uses Newton's Law of Universal Gravitation to propagate the position and velocity of the i^{th} satellite:

$$\begin{bmatrix} d\vec{r}_i/dt \\ d\vec{v}_i/dt \end{bmatrix} = \begin{bmatrix} \vec{v}_i \\ -\vec{r}_i\,\mu/\|\vec{r}_i\|^3 \end{bmatrix}.$$

Note that, in this case, future positions may be differentiated with respect to the initial orbital elements to obtain closed-form expressions for the gradient and hessian values used in optimization.

Non-Keplerian dynamics modify these equations to

$$\begin{bmatrix} d\vec{r}_i/dt \\ d\vec{v}_i/dt \end{bmatrix} = \begin{bmatrix} \vec{v}_i \\ \nabla\vec{U}_{t,i} + \vec{a}_{t,i} - \vec{r}_i\,\mu/\|\vec{r}_i\|^3 \end{bmatrix},$$

- $\vec{U}_{t,i}$ is the potential field arising from conservative, non-Keplerian forces.
- $\vec{a}_{t,i}$ is the net acceleration due to non-conservative, non-Keplerian forces.

Note that both $\vec{U}_{t,i}$ and $\vec{a}_{t,i}$ are evaluated at time t.

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Review of Prior Discussion

If we, for now, assume that **all forces acting on the satellite are conservative** (i.e., $\vec{a}_{t,i} = 0$), then we may use the dynamics

$$\begin{bmatrix} d\vec{r}_i/dt \\ d\vec{v}_i/dt \end{bmatrix} = \begin{bmatrix} \vec{v}_i \\ \nabla \mathcal{U}_{t,i} - \vec{r}_i\, \mu/\|\vec{r}_i\|^3 \end{bmatrix}.$$

Let us further assume that $\mathcal{U}_{t,i}$ may be fully expressed as a function of the **instantaneous values of the following orbital elements**:

1. Semi-major axis $a_{t,i}$;
2. eccentricity $e_{t,i}$;
3. right ascension of the ascending node (RAAN) $\Omega_{t,i}$;
4. inclination $I_{t,i}$;
5. argument of periapse $\omega_{t,i}$;
6. mean anomaly $M_{t,i}$;

and time t.

Review of Prior Discussion

Lagrange's Planetary Equations thus yield

$$\frac{da_{t,i}}{dt} = \frac{2}{n_{t,i}a_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial M_{t,i}} \qquad \frac{dI_{t,i}}{dt} = \frac{\cot(I_{t,i})}{n_{t,i}a_{t,i}^2 f_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial \omega_i} - \frac{f_{t,i}}{n_{t,i}a_{t,i}^2 \sin(I_{t,i})}\frac{\partial \mathcal{U}_{t,i}}{\partial \Omega_{t,i}}$$

$$\frac{de_{t,i}}{dt} = \frac{f_{t,i}^2}{n_{t,i}a_{t,i}^2 e_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial M_{t,i}} - \frac{f_{t,i}}{n_{t,i}a_{t,i}^2 e_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial \omega_{t,i}} \qquad \frac{d\omega_{t,i}}{dt} = \frac{f_{t,i}}{n_{t,i}a_{t,i}^2 e_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial e_{t,i}} - \frac{\cot(I_{t,i})}{n_{t,i}a_{t,i}^2 f_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial I_{t,i}}$$

$$\frac{d\Omega_{t,i}}{dt} = \frac{1}{n_i a_i^2 \sin(I_{t,i}) f_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial I_{t,i}} \qquad \frac{dM_{t,i}}{dt} = n_{t,i} - \frac{2}{n_{t,i}a_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial a_{t,i}} - \frac{f_{t,i}^2}{n_{t,i}a_{t,i}^2 e_{t,i}}\frac{\partial \mathcal{U}_{t,i}}{\partial e_{t,i}}$$

where $n_{t,i} = (\mu/a_{t,i}^3)^{1/2}$ and $f_{t,i} = (1 - e_{t,i}^2)^{1/2}$.

- Remark on singular cases:
  - $I_{t,i} = 0 \Longrightarrow \Omega_{t,i} = \text{undefined}$
  - $e_{t,i} = 0 \Longrightarrow \omega_{t,i} = \text{undefined}$
  - Different representations available for the singular cases.

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Review of Prior Discussion

Nonconservative forces are introduced by setting

$$\frac{\partial \mathcal{U}_{t,i}}{\partial *} = \frac{\partial \vec{r}_i}{\partial *} \cdot \vec{a}_{t,i}.$$

This enables inclusion of:

- Atmospheric drag;
- Solar radiation pressure;
- Thrust actuation;

Thrust actuation may also be used as a control term. Bounding this term is tantamount to bounding the level of control effort required to maintain swarm geometry.

Note that the bounds of some gravitational potentials can be determined:

Source	$\min(u_i)\ \left[\frac{km^2}{s^2}\right]$	$\max(u_i)\ \left[\frac{km^2}{s^2}\right]$	$\min(\lVert\nabla u_i\rVert)\ \left[\frac{km}{s^2}\right]$	$\max(\lVert\nabla u_i\rVert)\ \left[\frac{km}{s^2}\right]$
J_2	-3.02×10^{-2}	$+6.04 \times 10^{-2}$	3.769×10^{-9}	2.776×10^{-5}
☉	$-9.02 \times 10^{+2}$	$-8.72 \times 10^{+2}$	5.736×10^{-6}	6.125×10^{-6}
☿	-2.68×10^{-4}	-1.02×10^{-4}	4.701×10^{-13}	3.266×10^{-12}
♀	-8.22×10^{-3}	-1.25×10^{-3}	4.820×10^{-12}	2.078×10^{-10}
☾	-1.63×10^{-2}	-1.07×10^{-2}	2.318×10^{-8}	5.450×10^{-8}
♂	-7.68×10^{-4}	-1.07×10^{-4}	2.681×10^{-13}	1.377×10^{-11}
♃	-2.14×10^{-1}	-1.31×10^{-1}	1.360×10^{-10}	3.631×10^{-10}
♄	-3.16×10^{-2}	-2.30×10^{-2}	1.393×10^{-11}	2.630×10^{-11}
♅	-2.24×10^{-3}	-1.84×10^{-3}	5.821×10^{-13}	8.664×10^{-13}
♆	-1.59×10^{-3}	-1.46×10^{-3}	3.115×10^{-13}	3.680×10^{-13}

Motivation to Adopt a New Model

Why account for disturbances beyond the largest few?

- Prior formation analyses limited to **Low-N swarms**.
- Per the swarm initialization procedure that we have introduced in prior discussions, satellite states **are interconnected with one another**. We believe High-N swarms have the potential to display **chaotic behavior**.
  - It is an observable fact of nature that interconnected systems **tend towards chaos** as the system (and thus the number of connections) becomes larger.
  - **Chaotic systems are susceptible to variation in initial condition.**
- Even if High-N swarms do not display chaotic behavior, the dynamics are sufficiently nonlinear that **impact of individual terms is unknown – particularly on the relative motion.** Thus, we feel it necessary to test the impact of terms beyond the most common lower order terms.
- Besides the above, we do not see any compelling reason to deny the Air Force tools relevant **swarm-based applications that require high-fidelity dynamics models** if it so chooses (a goal in keeping with our longstanding goal of mission variability and customization)

Motivation to Adopt a New Model

- **Problem:** Our swarm analysis requires a way to test high-N swarms for sensitivity to small changes in the dynamics function without the ability to conduct true, on-orbit experiments.
- **Solution:** Obtain as close to exact model knowledge as possible using the IERS 2010 Conventions as a blueprint to construct a thorough dynamics model.
- **Benefits:** A framework wherein the dynamics function can be modified with additional terms; wherein **individual dynamics terms may be activated or deactivated** separately of one another to determine the impact they have on the evolution of the swarm over time.

Building the Dynamics Model

- We redefine the non-Keplerian dynamics by the relation

$$ {}_{\mathcal{G}}\dot{\vec{x}}_i = {}_{\mathcal{N}}\dot{\vec{x}}_i - {}_{\mathcal{N}}\dot{\vec{x}}_{\mathcal{G}} = \begin{bmatrix} {}_{\mathcal{N}}\vec{v}_i - {}_{\mathcal{N}}\vec{v}_{\mathcal{G}} \\ {}_{\mathcal{N}}\vec{a}_i - {}_{\mathcal{N}}\vec{a}_{\mathcal{G}} \end{bmatrix} = \begin{bmatrix} {}_{\mathcal{G}}\vec{v}_i \\ \vec{f}(t, {}_{\mathcal{G}}\vec{x}_i) - {}_{\mathcal{N}}\vec{a}_{\mathcal{G}}(t) \end{bmatrix}, $$

  where ${}_{\mathcal{G}}\vec{x}_i^T \overset{\text{def}}{=} \left[{}_{\mathcal{G}}\vec{r}_i^T \quad {}_{\mathcal{G}}\vec{v}_i^T\right]$ is the **orbital state** of the i^{th} satellite given position ${}_{\mathcal{G}}\vec{r}_i$ and velocity ${}_{\mathcal{G}}\vec{v}_i$ determined relative to the geocentric reference frame $\mathcal{G}$; ${}_{\mathcal{G}}\dot{\vec{x}}_i$ is its derivative with ${}_{\mathcal{G}}\vec{a}_i$ denoting acceleration.
- We define $\mathcal{G}$ (**commonly called "ECI"**) to be the set of coordinate axes whose origin is coincident with Earth's center of mass and whose axes are parallel to those of the J2000 inertial reference frame $\mathcal{N}$.
- We define $\mathcal{N}$ to be the set of coordinate axes whose origin is coincident with the solar system barycenter, **neglecting proper motion of the sun over mission-relevant timescales.**
- ${}_{\mathcal{N}}\vec{x}_{\mathcal{G}}^T \overset{\text{def}}{=} \left[{}_{\mathcal{N}}\vec{r}_{\mathcal{G}}^T \quad {}_{\mathcal{N}}\vec{v}_{\mathcal{G}}^T\right]$ is the orbital state of the Earth relative to $\mathcal{N}$.
- In general, ${}_{\mathcal{B}}\vec{x}_{\mathcal{A}}^T$ denotes the orbital state of **point or non-rotating reference frame** $\mathcal{A}$ as seen by an observer in reference frame $\mathcal{B}$.

Building the Dynamics Model

- $\vec{f}(t, {}_{\mathcal{G}}\vec{x}_i)$ contains higher-order dynamics terms.
- Models are available containing higher order terms which are dependent on Earth's orientation at time t.
  - Per the rules of transformation between rotating reference frames:

$$ {}_{\mathcal{E}}\vec{x}_i = \begin{bmatrix} \vec{r}_i \\ {}_{\mathcal{E}}\vec{v}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -({}_{\mathcal{G}}\vec{\boldsymbol{\omega}}_{\mathcal{E}}\times) & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \vec{r}_i \\ {}_{\mathcal{G}}\vec{v}_i \end{bmatrix} \stackrel{\text{def}}{=} {}_{\mathcal{E}}\boldsymbol{T}^{\mathcal{G}}\,{}_{\mathcal{G}}\vec{x}_i $$

  where ${}_{\mathcal{G}}\vec{\omega}_{\mathcal{E}}(t)$ is the angular velocity of $\mathcal{E}$ relative to $\mathcal{G}$, and ${}_{\mathcal{E}}\boldsymbol{T}^{\mathcal{G}}$ is the tensor which transforms a $\mathcal{G}$-relative state into an $\mathcal{E}$-relative state. $\boldsymbol{I}$ is the identity tensor.
  - Let ${}_{\mathcal{G}}\vec{a}_i^{\mathcal{E}}$ be the acceleration due to disturbances evaluated in frame $\mathcal{E}$.

$$ {}_{\mathcal{G}}\vec{a}_i^{\mathcal{E}} = \begin{bmatrix} ({}_{\mathcal{G}}\vec{\boldsymbol{\omega}}_{\mathcal{E}}\times)({}_{\mathcal{G}}\vec{\boldsymbol{\omega}}_{\mathcal{E}}\times) + ({}_{\mathcal{G}}\vec{\boldsymbol{\alpha}}_{\mathcal{E}}\times) & 2({}_{\mathcal{G}}\vec{\boldsymbol{\omega}}_{\mathcal{E}}\times) & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \vec{r}_i \\ {}_{\mathcal{E}}\vec{v}_i \\ {}_{\mathcal{E}}\vec{f}(t, {}_{\mathcal{E}}\vec{x}_i) \end{bmatrix} $$

$$ \Rightarrow {}_{\mathcal{G}}\vec{a}_i^{\mathcal{E}} \stackrel{\text{def}}{=} {}_{\mathcal{G}}\boldsymbol{A}^{\mathcal{E}}\,{}_{\mathcal{E}}\boldsymbol{T}^{\mathcal{G}}\,{}_{\mathcal{G}}\vec{x}_i + {}_{\mathcal{E}}\vec{f}(t, {}_{\mathcal{E}}\boldsymbol{T}^{\mathcal{G}}\,{}_{\mathcal{G}}\vec{x}_i) $$

Building the Dynamics Model

- We may similarly apply nonspherical Lunar gravity, which is defined in the Moon-centered, Moon-fixed frame $\mathcal{M}$.
- We will also denote Lunacentric coordinates $\mathcal{L}$ to be parallel to $\mathcal{N}$ with origin fixed to the Moon's center of mass.

  It follows that

$$_{\mathcal{L}}\vec{x}_i = {_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}} \Longrightarrow {_{\mathcal{M}}\vec{x}_i} = {_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right).$$

$$\Longrightarrow {_{\mathcal{L}}\vec{a}_i^{\mathcal{M}}} = {_{\boldsymbol{\mathcal{L}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right) + {_{\mathcal{M}}\vec{f}}\left(t, {_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right)\right).$$

- In a final step, we must account for the acceleration of $\mathcal{L}$ relative to $\mathcal{G}$.

$$_{\mathcal{G}}\vec{a}_i^{\mathcal{M}} = {_{\boldsymbol{\mathcal{L}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right) + {_{\mathcal{M}}\vec{f}}\left(t, {_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right)\right) + {_{\mathcal{G}}\vec{a}_{\mathcal{L}}}.$$

- It follows that ${_{\boldsymbol{\mathcal{L}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}} \equiv {_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}}$. Thus, we may write that

$$_{\mathcal{G}}\vec{a}_i^{\mathcal{M}} = {_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}}\,{_{\mathcal{G}}\vec{x}_i} + {_{\mathcal{M}}\vec{f}}\left(t, {_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right)\right) - \left({_{\mathcal{L}}\vec{a}_{\mathcal{G}}} - {_{\boldsymbol{\mathcal{L}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{L}}}}\,{_{\mathcal{L}}\vec{x}_{\mathcal{G}}}\right).$$

$$\Longrightarrow {_{\mathcal{G}}\vec{a}_i^{\mathcal{M}}} = {_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}}\,{_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}}\,{_{\mathcal{G}}\vec{x}_i} + {_{\mathcal{M}}\vec{f}}\left(t, {_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}}\left({_{\mathcal{G}}\vec{x}_i} - {_{\mathcal{G}}\vec{x}_{\mathcal{L}}}\right)\right) - {_{\mathcal{M}}\vec{a}_{\mathcal{G}}}.$$

Building the Dynamics Model

- A quick note from the previous slide concerning the expression

$$ {}_{\mathcal{G}}\vec{a}_i^{\mathcal{E}} = {}_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{E}}}\,{}_{\boldsymbol{\mathcal{E}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}\,{}_{\mathcal{G}}\vec{x}_i + {}_{\mathcal{E}}\vec{f}\left(t, {}_{\boldsymbol{\mathcal{E}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}\,{}_{\mathcal{G}}\vec{x}_i\right), $$

and

$$ {}_{\mathcal{G}}\vec{a}_i^{\mathcal{M}} = {}_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}\,{}_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}\,{}_{\mathcal{G}}\vec{x}_i + {}_{\mathcal{M}}\vec{f}\left(t, {}_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}\left({}_{\mathcal{G}}\vec{x}_i - {}_{\mathcal{G}}\vec{x}_{\mathcal{L}}\right)\right). $$

If we define

$$ {}_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{G}}} \stackrel{\text{def}}{=} {}_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{E}}}\,{}_{\boldsymbol{\mathcal{E}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}} + {}_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{M}}}\,{}_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}, $$

then the non-Keplerian dynamics encountered so far may be expressed as

$$ {}_{\mathcal{G}}\dot{\vec{x}}_i = \begin{bmatrix} {}_{\mathcal{G}}\vec{v}_i \\ {}_{\mathcal{E}}\vec{f}\left(t, {}_{\boldsymbol{\mathcal{E}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}\,{}_{\mathcal{G}}\vec{x}_i\right) + {}_{\mathcal{M}}\vec{f}\left(t, {}_{\boldsymbol{\mathcal{M}}}\boldsymbol{T}^{\boldsymbol{\mathcal{G}}}\left({}_{\mathcal{G}}\vec{x}_i - {}_{\mathcal{G}}\vec{x}_{\mathcal{L}}\right)\right) + {}_{\mathcal{G}}\vec{f}\left(t, {}_{\mathcal{G}}\vec{x}_i\right) + {}_{\boldsymbol{\mathcal{G}}}\boldsymbol{A}^{\boldsymbol{\mathcal{G}}}\,{}_{\mathcal{G}}\vec{x}_i - {}_{\mathcal{M}}\vec{a}_{\mathcal{G}} - {}_{\mathcal{N}}\vec{a}_{\mathcal{G}} \end{bmatrix}. $$

Building the Dynamics Model

- Earth's angular velocity ${}_{\mathcal{G}}\vec{\omega}_{\mathcal{E}}(t)$, angular acceleration ${}_{\mathcal{G}}\vec{\alpha}_{\mathcal{E}}(t)$, and linear acceleration ${}_{\mathcal{N}}\vec{\alpha}_{\mathcal{G}}(t)$ are, for our purposes, determined from empirical data collected by NASA and made available through **JPL's CSPICE software** at https://naif.jpl.nasa.gov/naif/index.html.
- For Earth's gravity calculation, **SGG-UGM-2** (http://icgem.gfz-potsdam.de/), published to degree 2190 but truncated to degree 96.
- Lunar gravitational acceleration implemented using **sphericalRFM_MOON_2519** (http://icgem.gfz-potsdam.de/), published to degree 2519 but truncated to degree 60.
- **Set up but not implemented:**
  - Gravitational pull by the sun and other planets.
  - A program to acquire the published monthly GRACE-FO data with a curve fit to obtain non-tidal influences on Earth's gravitational potential.
  - Earth Ocean Tides 2011a to implement the gravitational potential caused by Earth's ocean tides.
  - Atmosphere and Ocean De-Aliasing Level-1B to implement the gravitational potential of Earth's atmosphere.
  - International Earth Rotation and Reference System Service (IERS) guidelines on the following:
    - Gravitational potential caused by solid Earth tides.
    - Gravitational potential caused by solid earth and oceanic polar tides.
    - General relativistic corrections to dynamics (Schwarzschild-Lens-Thinning term).

Next Steps

- Implement the remaining IERS 2010 dynamics terms.
- Apply Tschauner-Hempel equations.
- Include terms for satellite-applied controls (e.g., thrust).
- Determine bounds of control effort required to maintain geometry.
- Satellite networked architecture.