Global Search Approach to Underactuated Docking Operations Via Model Predictive Control and the Cross-Entropy Method

Assured Autonomy in Contested Environments (AACE) Spring 2024 Review Agenda

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NSF

SSG Space Systems Group

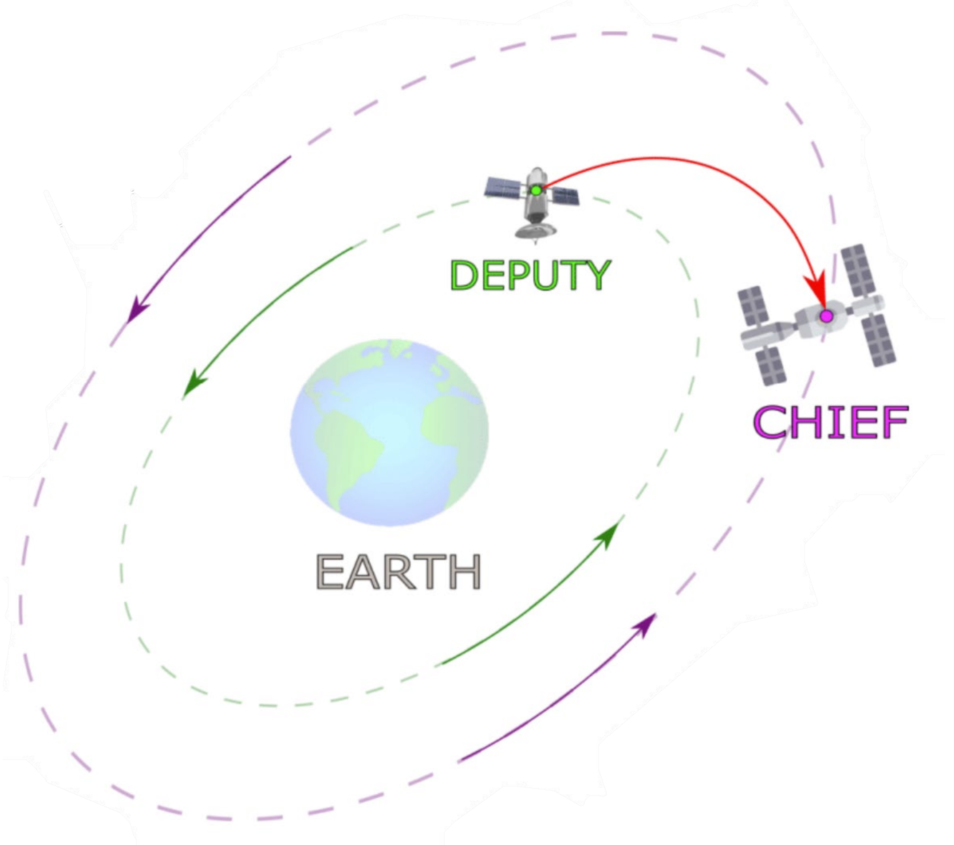
2024 Accomplishments

- 4th collaborative year with AFRL (RV)
- Transition from NSF GRFP to SMART
- Publications
    - A. Aborizk, N. G. Fitz-Coy, “Multiphase Autonomous Docking via Model-Based and Hierarchical Reinforcement Learning”, *Journal of Spacecraft and Rockets, 2024*
    - A. Aborizk, N. G. Fitz-Coy, A. Soderlund, “3d Underactuated Spacecraft Docking using Legendre Gauss Radau Collocation”, *IEEE Aerospace Conference,* Big Sky, 2024
    - A. Aborizk, A. Soderlund, N. G. Fitz-Coy, “An On-Line Global Search Approach to Underactuated Docking Operations Via Model Predictive Control and the Cross-Entropy Method”, *International Symposium of Space Flight Dynamics,* Darmstadt, 2024

Motivation

Goals of the benchmark problem

- Autonomy
  - Proliferation of spacecraft in orbit
  - Overburdened ground control operators
  - Advancing a critically needed space-based technology
- Weight reduction
  - Small satellites are becoming more prevalent
  - Lower cost and lightweight
  - Removal of superfluous actuators can reduce mass and/or allow more room for scientific instrumentation
- Fault tolerance
  - Some satellites experience actuator faults during the launch process
- The autonomous rendezvous, proximity operations, and docking (ARPOD) field seeks to enable technologies like on-orbit satellite servicing, refueling, and constellations management
- This research provides an on-line global search solution to a benchmark problem introduced by researchers at the U.S. AirForce Research Laboratory



Depiction of a docking maneuver [3]

The Case Study

The goal of this case study is to conjoin two orbiting spacecraft

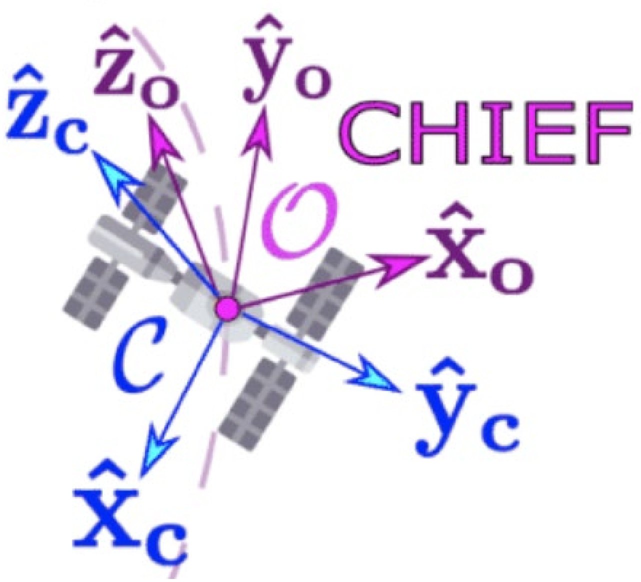
- The chief is passive and cooperative
- The deputy is active but underactuated

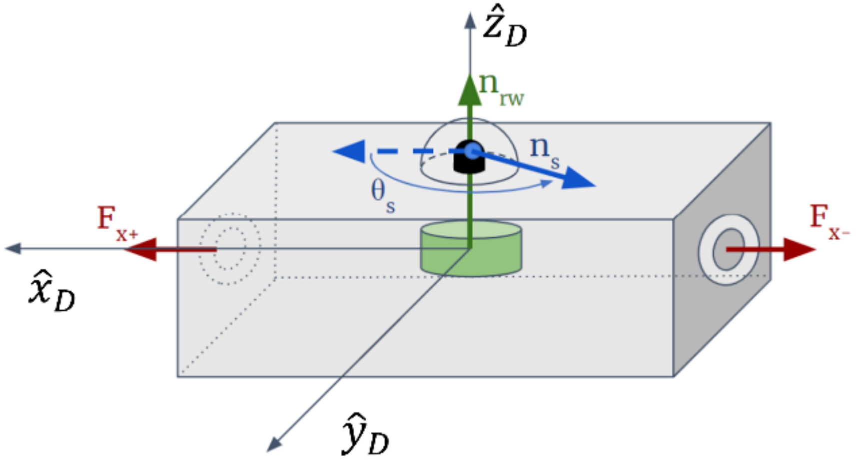
Underactuated

- The deputy has thrust about a unilateral axis
- Attitude is constrained to the $\hat{\boldsymbol{Z}}_D$-axis via a gimbaled sensor
- Attitude is controlled by a flywheel

Reference Frames

- Reference frame $\mathcal{O}$ is fixed to the center of mass of the Chief
- Frame $\mathcal{C}$ aligns with the principal axes of the Chief
- Frame $\mathcal{D}$ aligns with the principal axes of the Deputy





Chief reference frame (top) [3]. Deputy reference frame (bottom)[2]

The Dynamics are Coupled!

- Let the control-input be defined as

$$\boldsymbol{u}_{\boldsymbol{d}}^{\mathcal{D}} = \left(F_{x}, \dot{\psi}\right)^{T}$$

- The entire state space is defined as

$$\mathbf{x} \coloneqq \left(\delta x, \delta y, \theta, \delta\dot{x}, \delta\dot{y}, \dot{\theta}\right)^{T} \in R^{6}$$

$$\dot{\boldsymbol{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\theta)\boldsymbol{u}_{\boldsymbol{d}}^{\mathcal{D}}$$

- Flywheel actuation is used to calculated angular acceleration using

$$\ddot{\theta} = -\frac{D\dot{\psi}}{I_z}$$

- Coupling occurs in the control

$$F_{xy} = \begin{bmatrix} cos(\theta) \\ sin(\theta) \end{bmatrix} F_x$$

- where,

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\eta^2 & 0 & 0 & 0 & 2\eta & 0 \\ 0 & 0 & 0 & -2\eta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{B}(\theta) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\cos(\theta)}{m_c} & 0 \\ \frac{\sin(\theta)}{m_c} & 0 \\ 0 & \frac{-D}{I_z} \end{pmatrix}$$

The Problem Cannot Be Solved with Classical Control

Let the deputy be defined as

$$x \coloneqq (\delta x, \delta y, \delta \dot{x}, \delta \dot{y}, \theta, \dot{\theta})^T \in R^6$$

The underactuated ARPOD problem is **solved** if, given prescribed state constraint set $\mathcal{X} \subseteq \mathbb{R}^6$ and input constraint set $\mathcal{U} \subseteq \mathbb{R}^2$, a control trajectory $u(t): \mathbb{R}_{\geq 0} \rightarrow \mathcal{U}$ can be found such that $\mathrm{x} \in \mathcal{X}\ \forall\, t \geq 0$ and $\mathrm{x} \rightarrow 0$ as $t \rightarrow \infty$.

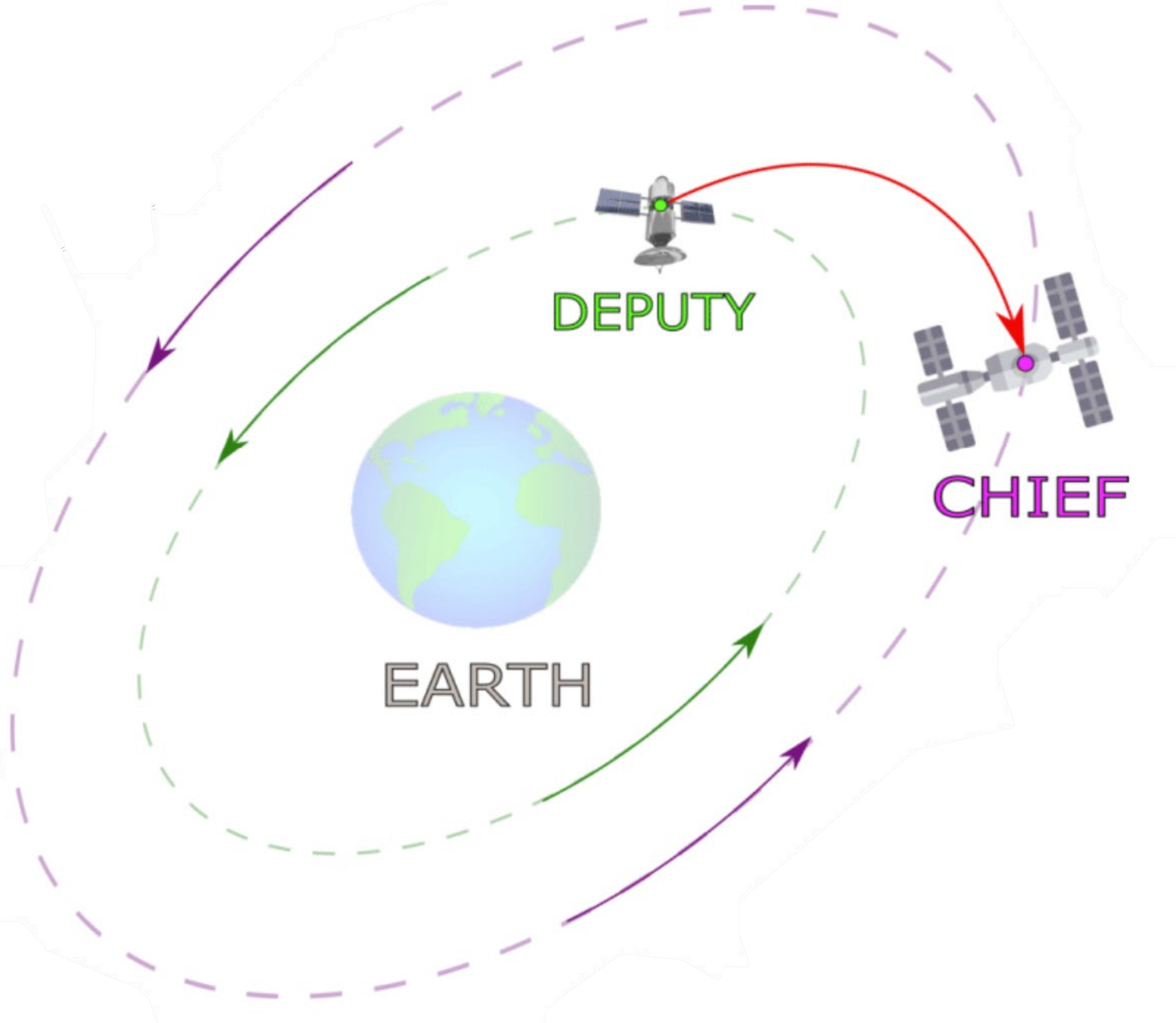
When linearized about the equilibrium point $x^* = 0$, the system is not controllable

$$\mathcal{C} = [B(0)\ AB(0) \ldots A^5 B(0)]$$

Additionally, linearization about the origin shows it is not stabilizable

$$rank\ ([\lambda_l I\ - A\ B(0)]) = n$$

$$\forall Re(\lambda_l) \geq 0, \qquad l = 1,2, \ldots, 6$$



Depiction of a docking maneuver [3]

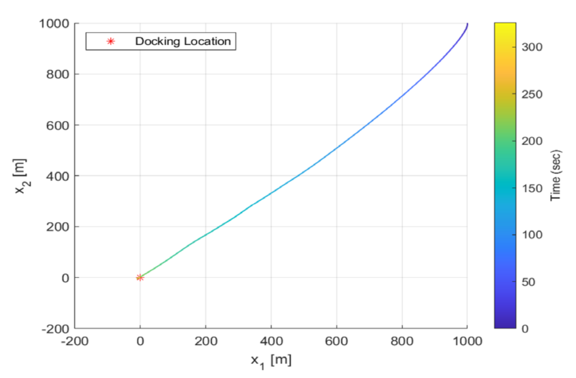
Stability Through Numerical Approaches

- Local asymptotic stability was determined feasible via hybrid methods through analyses done in geometric control, Floquet theory, and homogeneity [3]

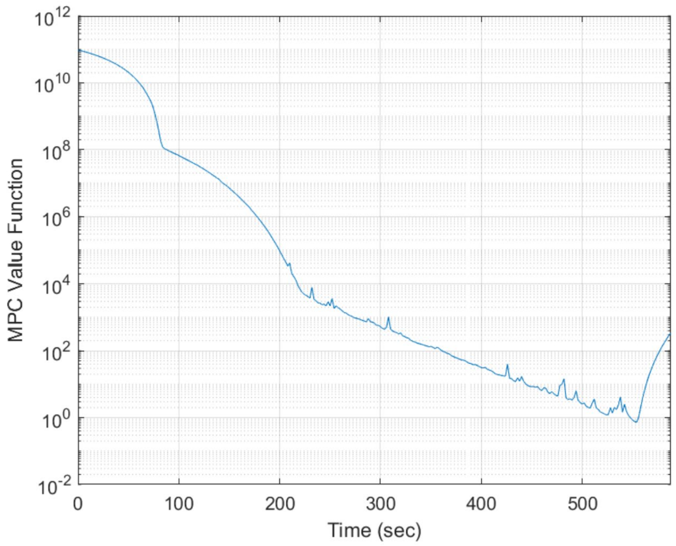
- However, numerical approaches have experienced challenges

- nMPC was used to solve this problem in [4] but was unable to produce a monotonically decreasing cost function especially near the origin

$$\forall x, y \qquad s.t.\ x \leq y, \qquad f(x) \leq f(y)$$



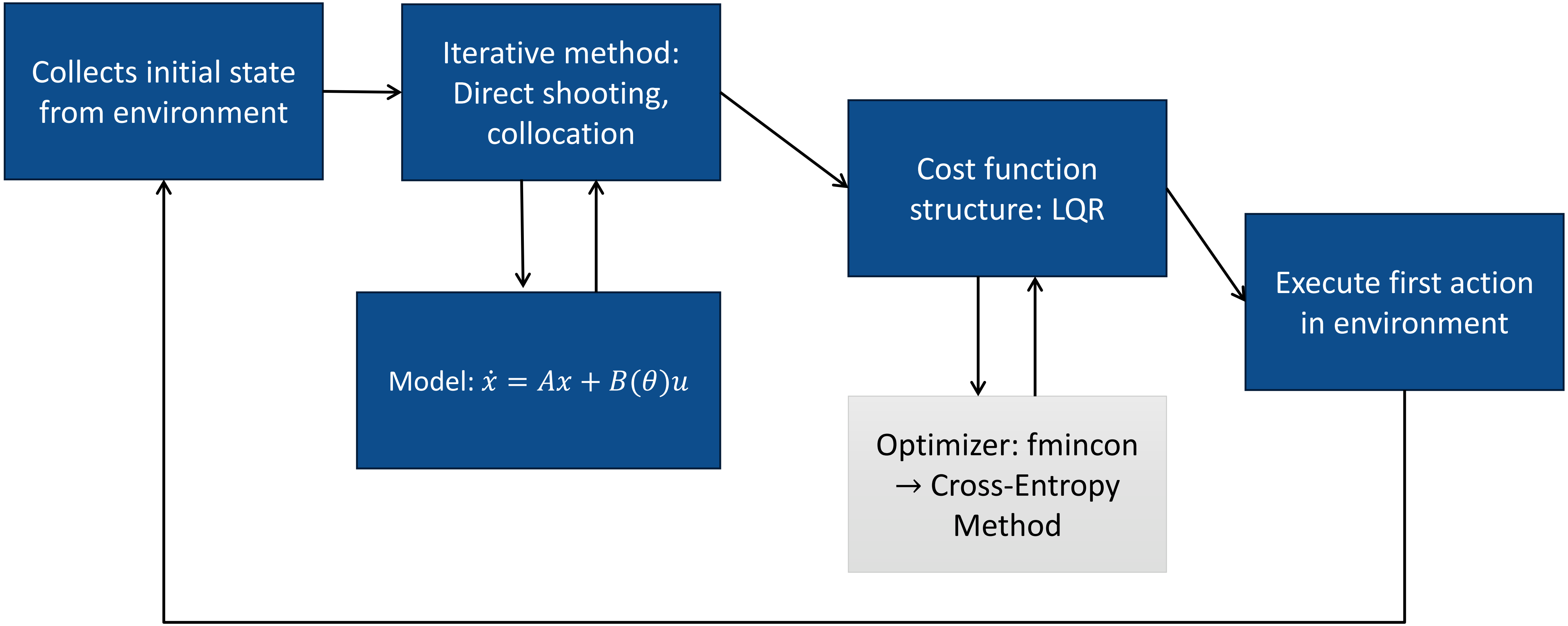
(a) The chaser satellite trajectory in Hill's frame.



nMPC trajectory presented in [4]

[4] A. Zaman, A. Soderlund, C. Petersen, S. Phillips, "Autonomous Satellite Rendezvous and Proximity Operations via Model Predictive Control Methods," in AAS/AIAA Astrodynamics Specialist Conference, Big Sky, 2021.

nMPC vs CEMPC



Cross-Entropy Method Model Predictive Control (CEMPC)

Methodology: The Cross-Entopy Method (CEM)

- CEM is a ***gradient-free optimization strategy*** that produces a ***global search*** of the state space during optimization [12].
- It typically lend itself well to bypassing shallow local optima with a higher probability than most gradient-based solvers

How it works

- Perform random shooting
- Select the J highest scoring action sequences
- utilizes them to compute a multivariate mean, μ, and covariance, Σ of the Gaussian trajectory distribution

$$\boldsymbol{\mu}_{t'}^{m+1} = \alpha * mean(\boldsymbol{A}_{elites}) + +(1-\alpha)\boldsymbol{\mu}_{t'}^{m}$$

$$\boldsymbol{\Sigma}_{t'}^{m+1} = \alpha * variance(\boldsymbol{A}_{elites}) + (1-\alpha)\boldsymbol{\Sigma}_{t'}^{m}$$

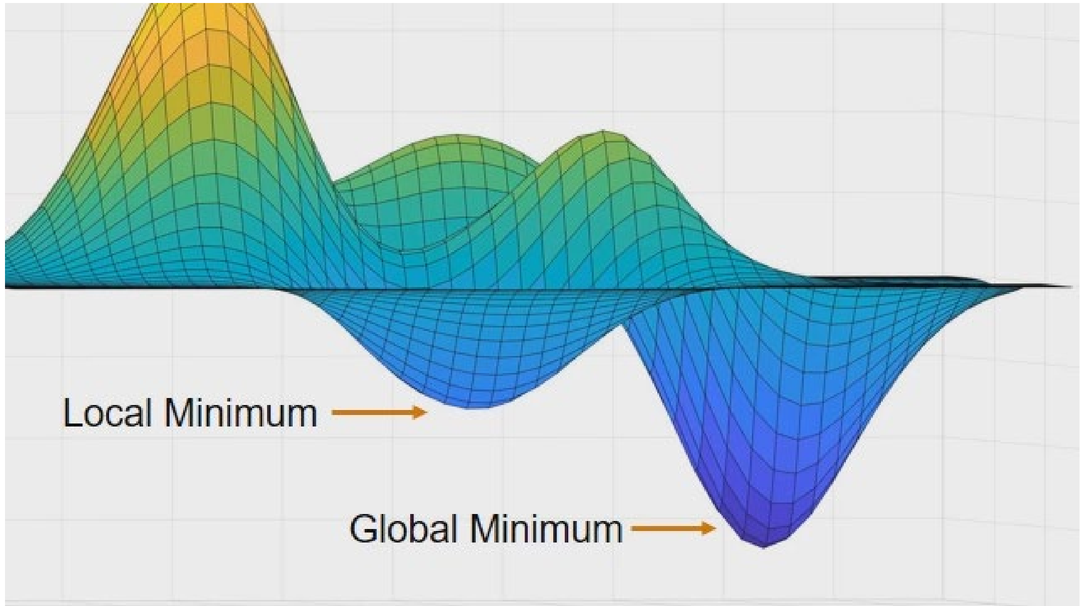
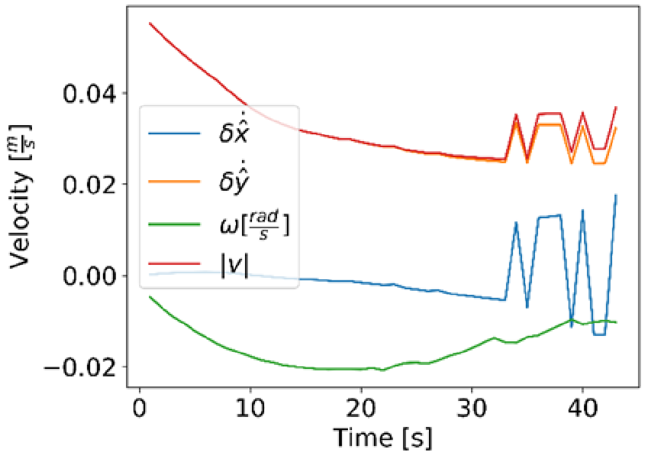
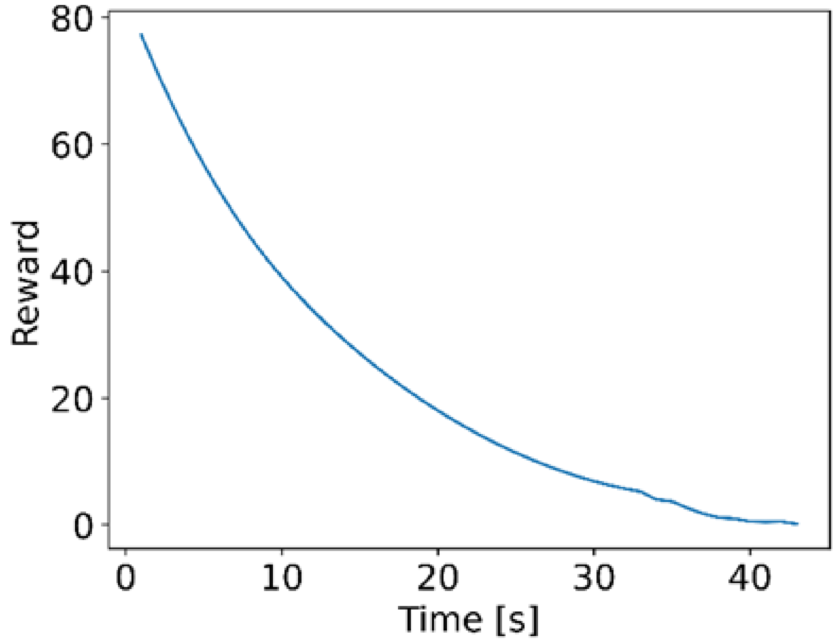
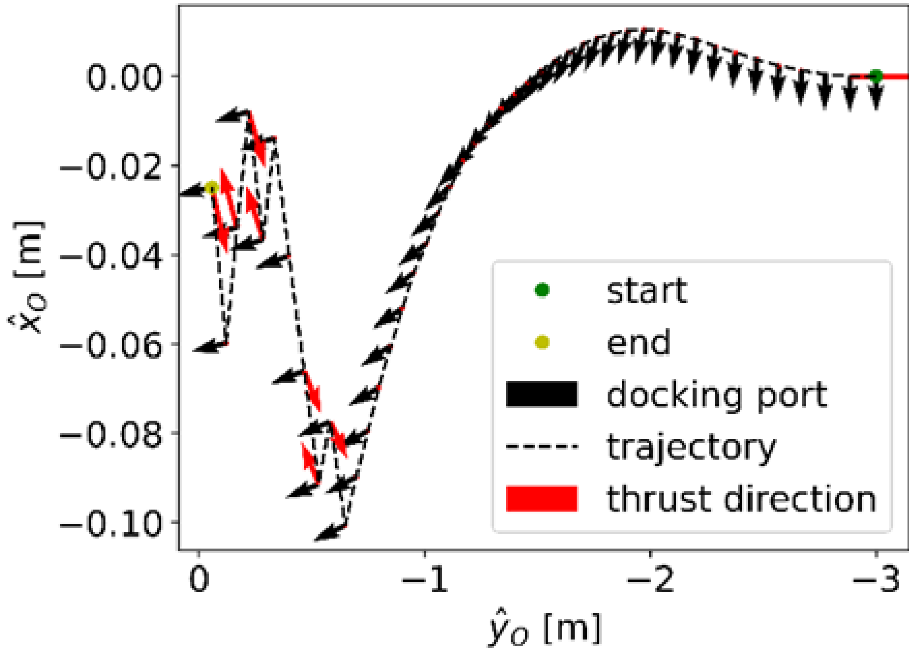


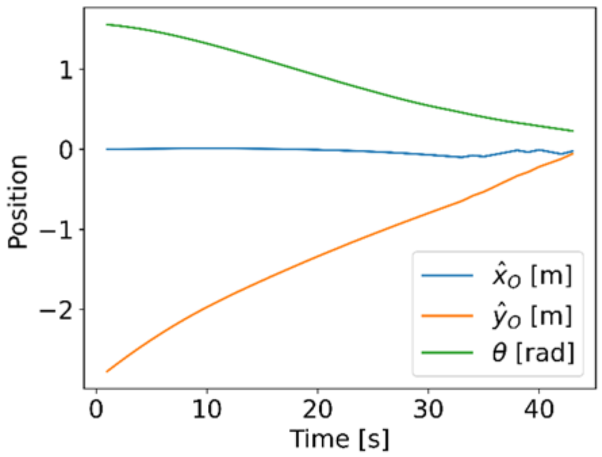
Image credit, MathWorks

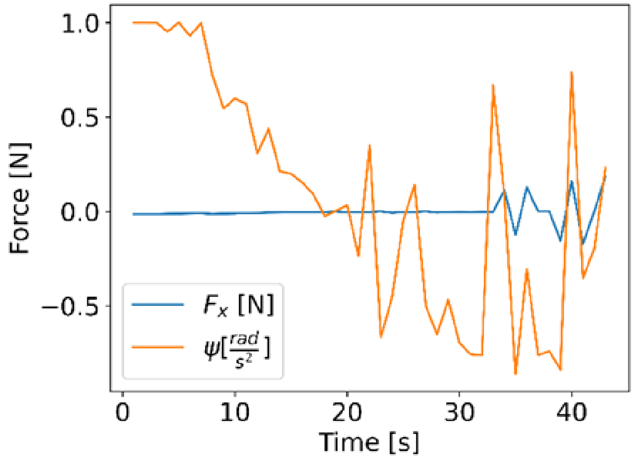
Stability/Feasibility Near Equilibrium





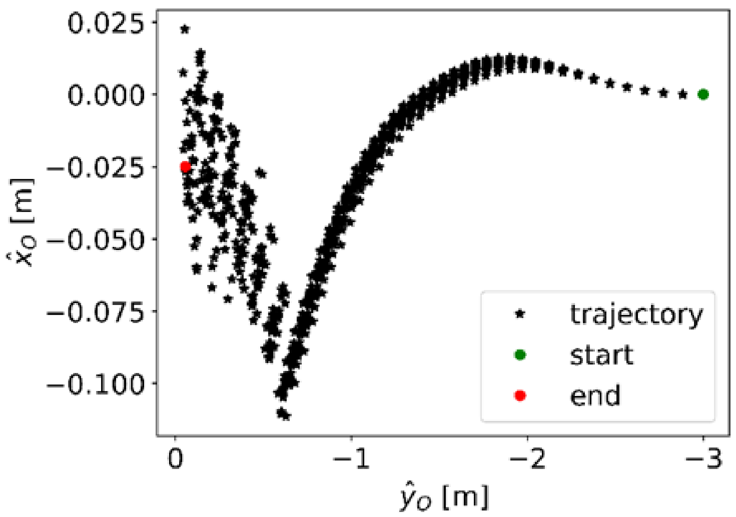




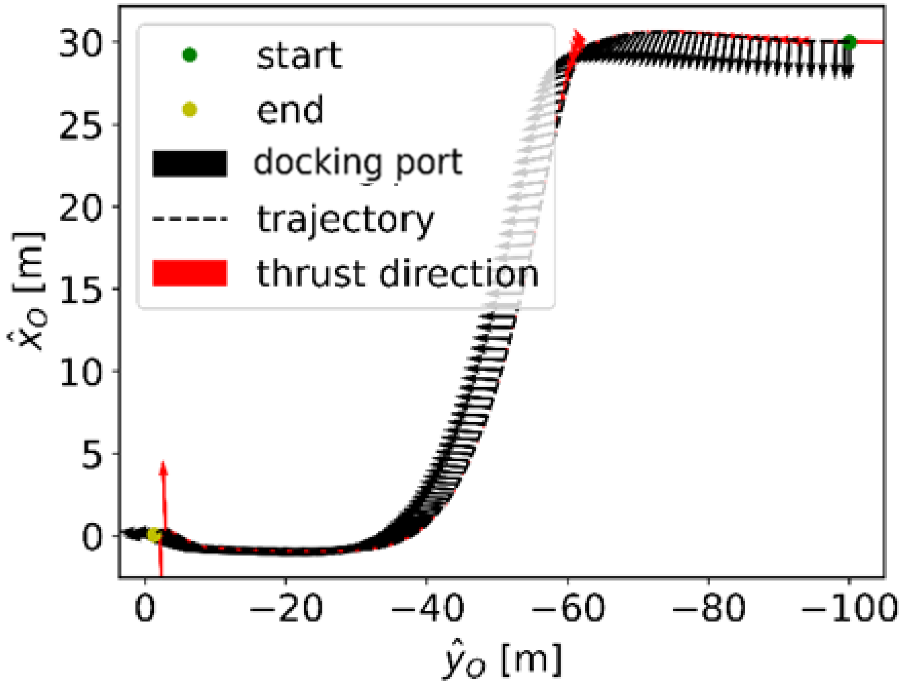


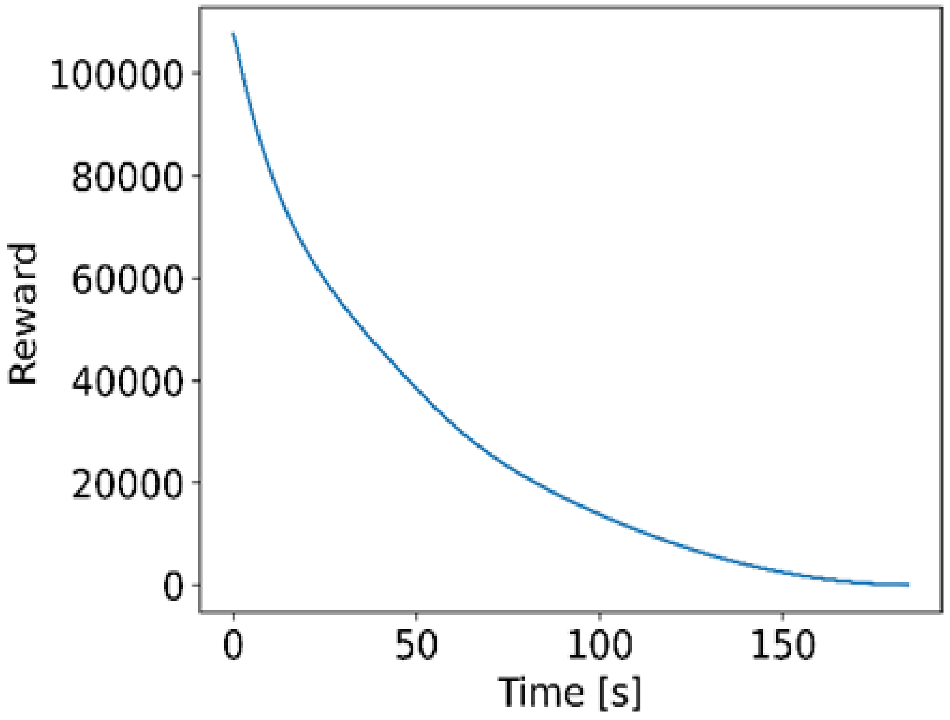
Numerical Stability

Not only is monotonicity achievable with this algorithm but trajectories are reproducible with a variable seed.



Analysis of Numerical Stability and Over Longer Time Periods





More Stability

INT. J. CONTROL, 1995, VOL. 62, NO. 5, 1217–1229

Receding horizon control and discontinuous state feedback stabilization

EDWARD S. MEADOWS†, MICHAEL A. HENSON‡§, JOHN W. EATON† and JAMES B. RAWLINGS†||

This paper addresses three aspects of receding horizon control in discrete-time: (1) feedback stabilization of general nonlinear systems with receding horizon control; (2) the generation of stabilizing feedback control laws that are discontinuous in the state; and (3) the suitability of receding horizon control to stabilize feedback-linearizable systems. The nonlinear receding horizon controller is shown via a Lyapunov function argument to be asymptotically stabilizing for a large class of nonlinear systems. As a special case, nonlinear systems that can be locally feedback linearized can be locally asymptotically stabilized with nonlinear receding horizon control. A simple example shows that there exist controllable nonlinear discrete-time systems that cannot be asymptotically stabilized with continuous feedback. For this example, the nonlinear receding horizon controller generates an asymptotically stabilizing feedback law. The discontinuity in the resulting feedback law is discussed and numerical results are provided.

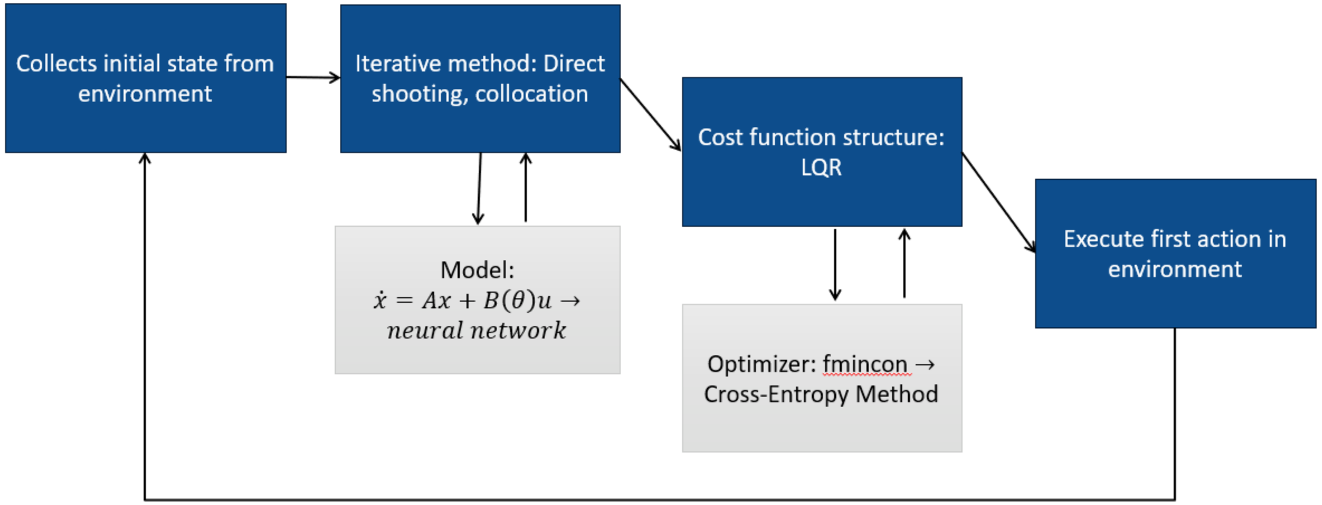
Stability

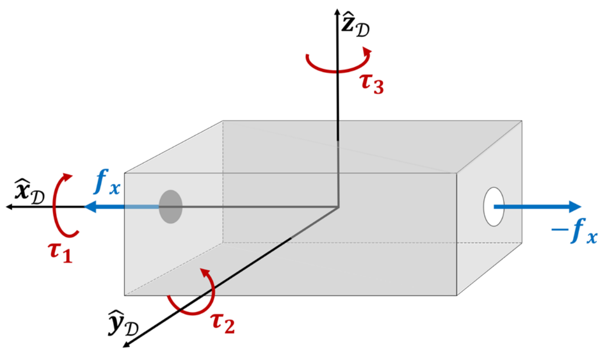
- Nonlinear, deterministic, discrete time system
- The point $\mathbf{0_8}$ must be contained in $\mathcal{X} \times \mathcal{U}$
- $\mathcal{X} \times \mathcal{U}$ must be closed and continuous
- Let $L(x, u)$ be the stage cost
  - L is continuous, $L(0, u) \to \infty$ as $||u|| \to \infty$
  - This implies the existence of a bounded open-loop optimal control sequence
  - L(0,0)
  - There exists non-decreasing $\gamma: [0, \infty) \to [0, \infty)$ such that $\gamma(0) = 0$ and $0 < \gamma(||x, u||) \leq L(x, u)$ for all $(x, u) \neq (0,0)$

- If the optimal cost, Φ_N^*, is continuous at the origin, x=0, and L satisfies the conditions above then the origin is an asymptotically stable equilibrium point

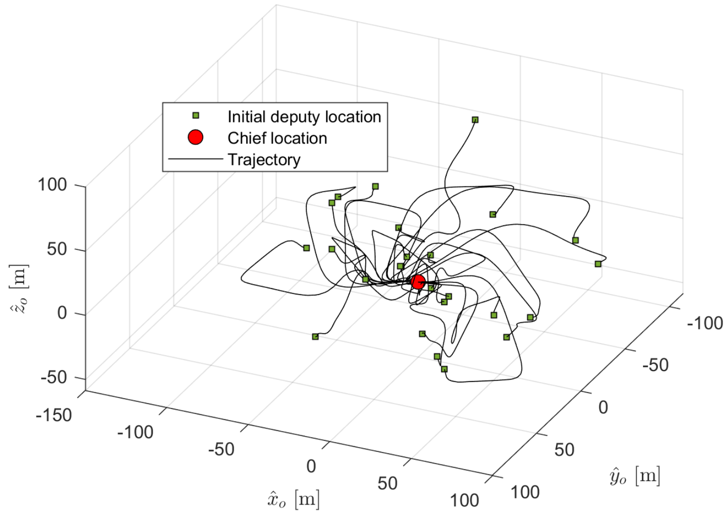
Future Work

- Expand CEMPC to a Model-Based Reinforcement Learning Algorithm to Enable System Identification
- Expand the simulation to 6 degrees of freedom to account for $\hat{\boldsymbol{z}}_O$-axis effects
- Explore stability and convergence proofs further
- Explore methods to improve robustness to initial conditions
- Explore methods to improve computation time





Orientation capabilities have been expanded in [6] using MRPs



Questions

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