

Robust Control via Adversarial Training

Hao-Lun Hsu

Miroslav Pajic

CPSL@Duke

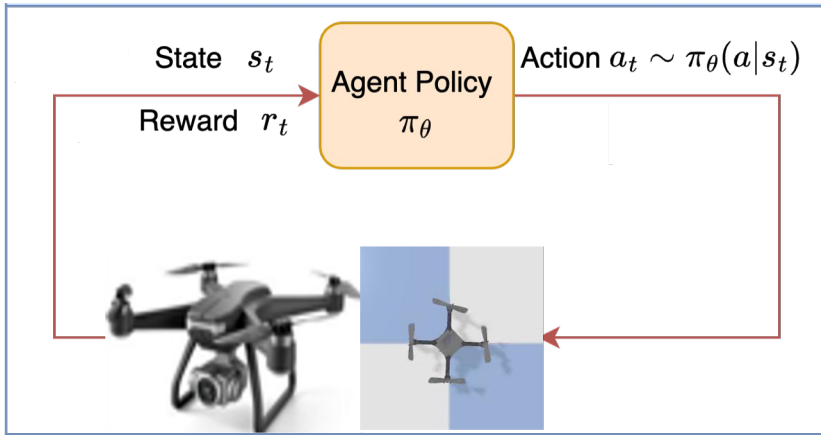
Department of Electrical and Computer Engineering

Department of Computer Science

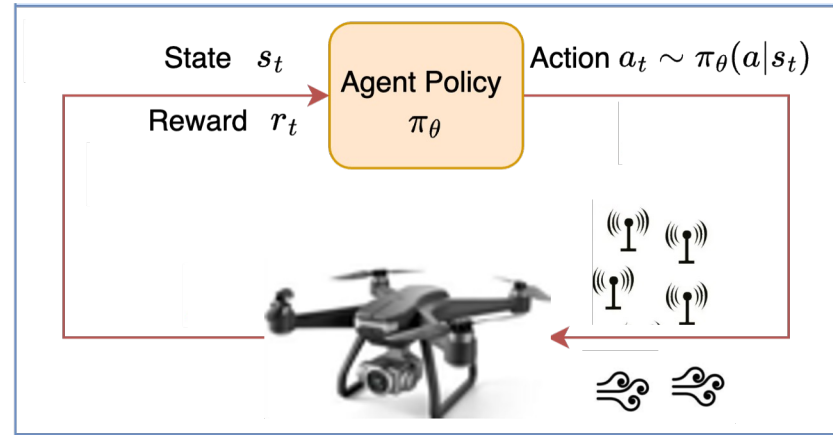
Department of Mechanical Engineering and Material Science

Duke University

Training



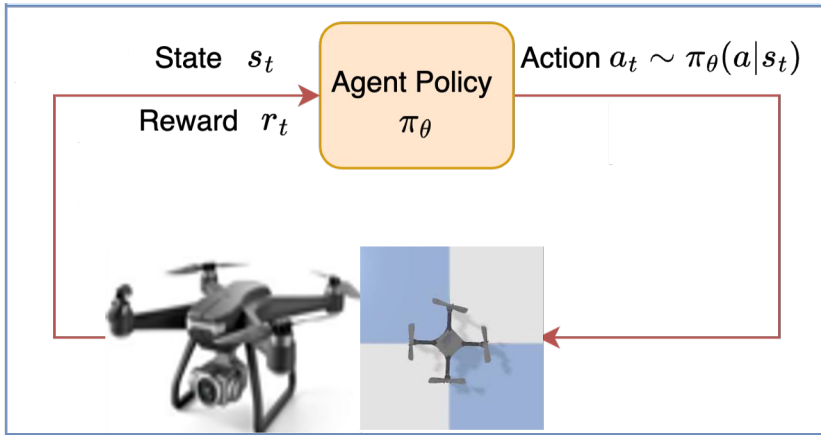
Testing



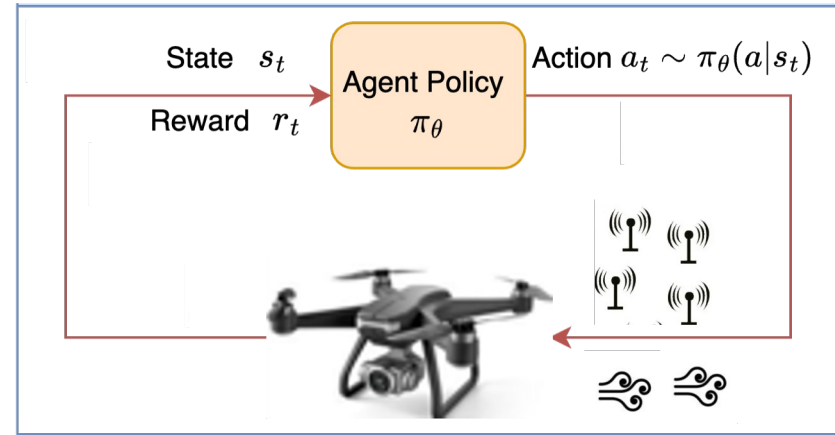
1. Train in **real world**: expensive, dangerous, and time-intensive \rightarrow a limit set of training scenarios
2. Train in **simulation**: Sim-to-Real gap (reality of simulation) \rightarrow not robust to modeling errors

Introduction

Training

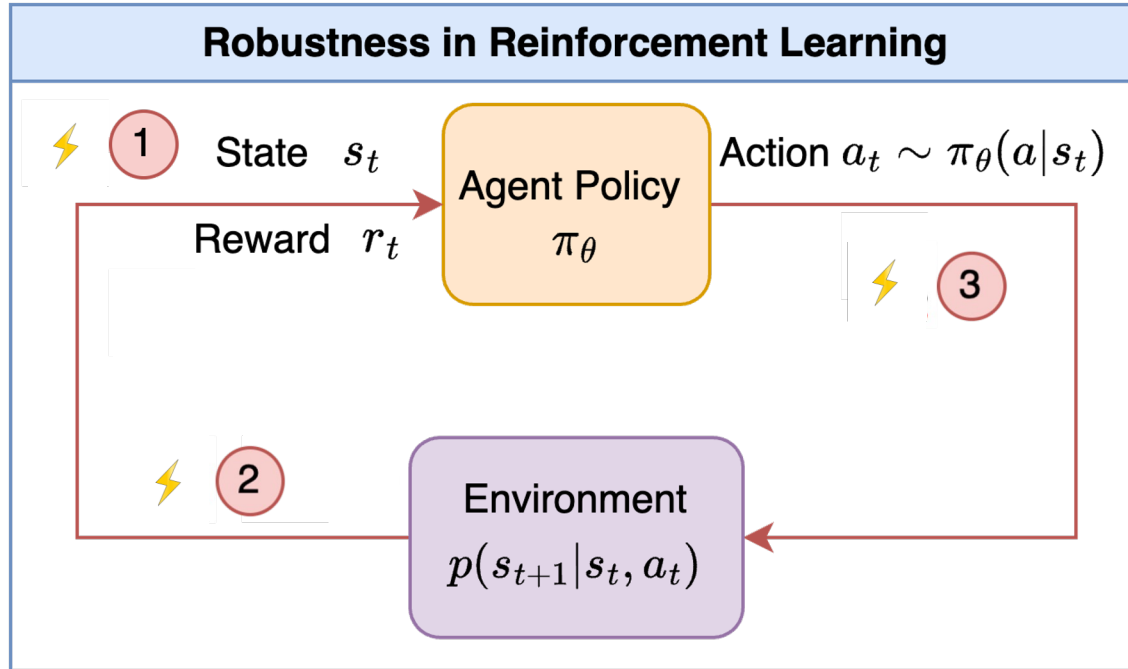


Testing



Robust RL takes the uncertainty of *internal parameters* and *external disturbances* into account

Motivation: Robust Control



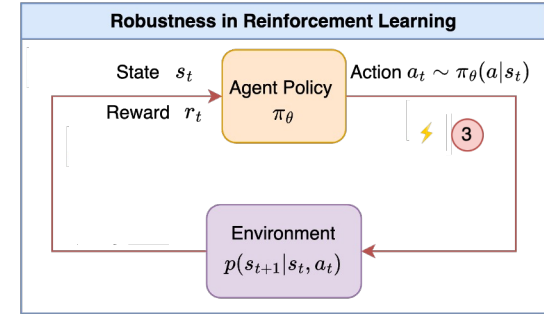
Sources of uncertainty/errors:

1. **Sensing:** observed states may be different from the true states
2. **Modeling errors:** Transitions dynamics may change
3. **Actuation:** Applied actions may be different from the agent's intention

Robust Control Design with 2-Player Game Design

$$R(\theta, \phi) \doteq \mathbb{E}_{s_0 \sim p_0} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t^p, a_t^a) \right]$$

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi)$$



Pros

1. Optimize the worst-case performance of RL agents under disturbance
2. Empirical success

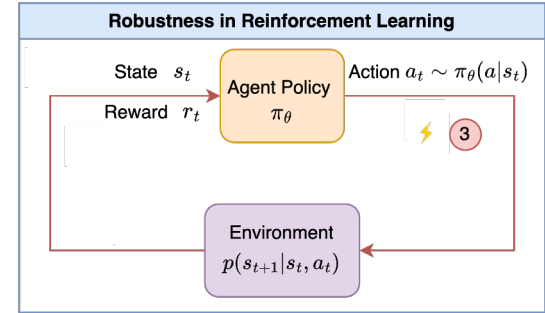
Cons

1. Inner minimization problem is difficult to solve \rightarrow **local-optimum**
2. worst-case optimization can result in **over-conservation** if adversary is overly capable

Robust Control Design with 2-Player Game Design

$$R(\theta, \phi) \doteq \mathbb{E}_{s_0 \sim p_0} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t^p, a_t^a) \right] \quad (1)$$

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi) \quad (2)$$



[NeurIPS24*] **Adversarial herding** for better approximation of the optimal adversary

[ICRA24] **Adaptive adversary** for unknown adversary strength

$$R(\theta, \phi_i) \doteq \mathbb{E}_{s_0 \sim p_0} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, (1 - \alpha)a_t^p + \alpha a_t^a) \right] \mathcal{C}$$

[L4DC24] **Efficient exploration** via Langevin Monte Carlo with robustness

1. J Dong* and HL Hsu* et al., "Robust Reinforcement Learning through Efficient Adversarial Herding", under review, 2024.
2. HL Hsu et al., "REFORMA: Robust REinFORceMent Learning via Adaptive Adversary for Drones Flying under Disturbances" in *IEEE International Conference on Robotics and Automation (ICRA)*, 2024.
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Robust Control Design with 2-Player Game Design

1. **Adversarial ensemble** which involves a group of adversaries [1]
 - a. Special case in noisy action robust MDP: Adaptive adversary for unknown adversary strengths [2]
2. **Efficient exploration** via Langevin Monte Carlo with robustness [3]

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi)$$

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Robustness with Adversarial Ensembles

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi)$$

Update a **single adversary** with first-order optimization method to solve inner optimization

Robustness with Adversarial Ensembles

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi) \quad \rightarrow \quad \max_{\theta \in \Theta} \min_{\phi \in \hat{\Phi}} R(\theta, \phi)$$

Update a **single adversary** with first-order optimization method to solve inner optimization

Employ a set of fixed adversaries $\hat{\Phi} \doteq \{\phi_i\}_{i=1}^m$ where m is the total number of adversaries and for all $i \in [m]$, $\phi_i \in \Phi$

Robustness with Adversarial Ensembles

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The gradient of $R(\theta, \phi)$ with respect to the adversary's parameter is **d-dimensional**

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1-dimensional $R(\theta, \phi)$ needs to be approximated

Robustness with Adversarial Ensembles

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi)$$

$$\max_{\theta \in \Theta} \min_{\phi \in \hat{\Phi}} R(\theta, \phi)$$

Efficiently approximate?

Update a **single adversary** with first-order optimization method to solve inner optimization

Employ a set of fixed adversaries $\hat{\Phi} \doteq \{\phi_i\}_{i=1}^m$ where m is the total number of adversaries and for all $i \in [m]$, $\phi_i \in \Phi$

The gradient of $R(\theta, \phi)$ with respect to the adversary's parameter is **d-dimensional**

1-dimensional $R(\theta, \phi)$ needs to be approximated

Definition 1: For a function $h : \mathcal{X} \rightarrow \mathbb{R}$, we define its L^∞ norm as $\|h\|_\infty = \sup_{x \in \mathcal{X}} |h(x)|$

Definition 2: Let (\mathcal{U}, d) be a metric space where $d : \mathcal{U} \times \mathcal{U} \rightarrow \mathbb{R}^+$ is the metric function. Then a finite set $\mathcal{X} \subset \mathcal{U}$ is an ϵ -packing if no two distinct elements in \mathcal{X} are ϵ -close to each other, i.e.,

$$\inf_{x, x' \in \mathcal{X}: x \neq x'} d(x, x') > \epsilon.$$

Insights from the theoretical results

- When the adversaries in the ensemble are distinct to each other, the accuracy for approximating the true worst-case performance can be improved with increased number of adversaries
- Robust optimization with an adversary ensemble solves the initial optimization problem!

Upper Bound of Almost Sure Approximation

Let R_{Φ} denote a function class as $R_{\Phi} \doteq \{R_{\phi} \doteq R(\theta, \phi) : \Theta \rightarrow \mathbb{R} \mid \phi \in \Phi\}$.

→ The number of adversaries needed to approximate the inner optimization problem is in approximately **linear** order of the desired precision if the set of adversaries are different enough.

Assumption 1: Assume that R_{Φ} has finite radius under this metric, i.e., $\sup_{\phi, \phi' \in \Phi} d(R_{\phi}, R_{\phi'}) \leq r_{\max}$ where $r_{\max} < \infty$ is a finite number.

Interpretation of Assumption 1

- The performance of any protagonist policy in two different environments cannot vary infinitely
- The number of adversaries needs for approximation is about $O(\frac{1}{\epsilon})$

Theorem 1: Consider the metric space $(R_{\Phi}, \|\cdot\|_{\infty})$ where for any two functions $R_{\phi}, R_{\phi'} \in R_{\Phi}$, the distance between them is defined as $d(R_{\phi}, R_{\phi'}) \doteq \|R_{\phi} - R_{\phi'}\|_{\infty}$. With assumption 1, let $\hat{\Phi} = \{\phi_i\}_{i=1}^m \subset \Phi$, if $R_{\hat{\Phi}}$ is a maximal ϵ -packing then $|R_{\hat{\Phi}}| \geq \lceil \frac{r_{\max}}{\epsilon} \rceil$ so that

$$|R(\theta, \phi^*) - R(\theta, \hat{\phi})| \leq \epsilon$$

Upper Bound of Approximation with High Probability

Theorem 2: Assume that Φ is a metric space with a distance function $d : \Phi \times \Phi \mapsto \mathbb{R}$. Let σ be any probability measure on Φ . Let $\hat{\Phi} = \{\phi_i\}_{i=1}^m$ be a set of independently sampled elements from following identical measure σ . consider a fixed $\theta \in \Theta$ and assume that $R(\theta, \phi)$ is an L_ϕ -Lipschitz continuous function of with respect to the metric space (Φ, d) . Let $\hat{\phi}$ and ϕ^* be defined the same as in Theorem 1. For presentation simplicity, assume that $\sigma(\{\phi : d(\phi, \phi^*) \leq \epsilon\}) \geq L_\sigma \epsilon$. Let $0 < \delta < 1$ denote the probability of a bad event. Then with probability $1 - \delta$, the approximation error of $\hat{\phi}$ on the inner optimization problem is bounded by ϵ if $m \geq \log(\delta) \log^{-1}(1 - \frac{L_\sigma}{L_\phi} \epsilon)$

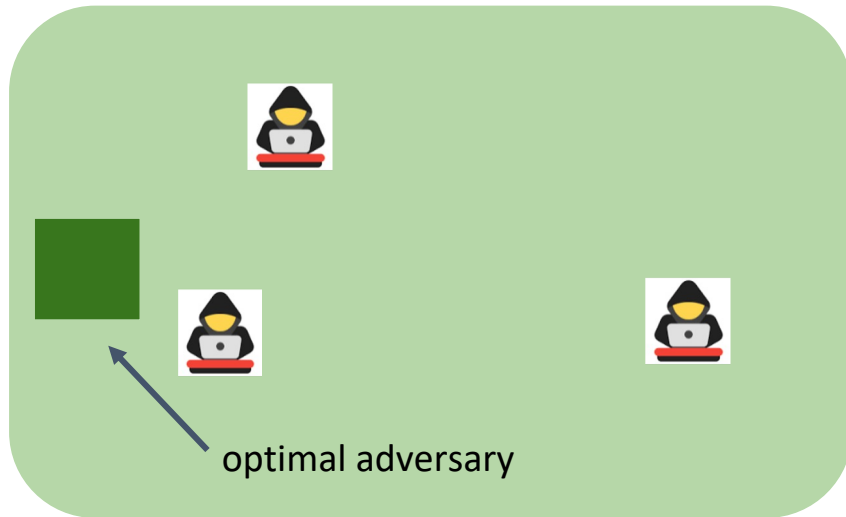
Now let $\phi_i \in \hat{\Phi}$ be learners (Φ is an adversary ensemble), instead of fixed adversaries.

$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi) \quad (2)$$

$$\max_{\theta \in \Theta} \min_{\phi_1, \dots, \phi_m \in \Phi} \min_{\phi \in \{\phi_i\}_{i=1}^m} R(\theta, \phi) \quad (3)$$

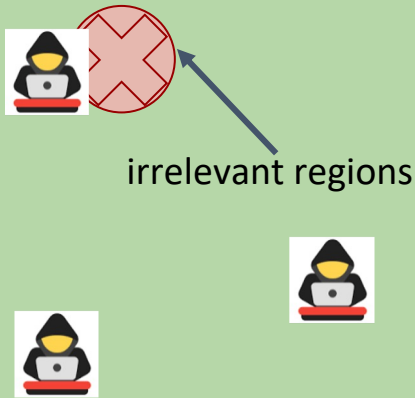
Lemma 1: The solution set to the optimization problem (2) is identical to the solution set of the optimization problem (3).

$$\max_{\theta \in \Theta} \min_{\phi_1, \dots, \phi_m \in \Phi} \min_{\phi \in \{\phi_i\}_{i=1}^m} R(\theta, \phi)$$



1. Efficient approximation of the inner optimization
i.e., the size of **adversary herd** is upper-bounded
to obtain sufficient approximation precision.

$$\max_{\theta \in \Theta} \min_{\phi_1, \dots, \phi_m \in \Phi} \frac{1}{|I_{\theta, \hat{\Phi}, k}|} \sum_{i \in I_{\theta, \hat{\Phi}, k}} R(\theta, \phi_i)$$



2. Resolving Potential Over-Pessimism

i.e., modify the objective from optimizing its worst-case performance, to optimizing its average performance over the **worst-k adversaries**

Adversarial Herd with Optimization Over *Worst-k* Adversaries

$$\max_{\theta \in \Theta} \min_{\phi_1, \dots, \phi_m \in \Phi} \frac{1}{|I_{\theta, \hat{\Phi}, k}|} \sum_{i \in I_{\theta, \hat{\Phi}, k}} R(\theta, \phi_i)$$

Algorithm 1 ROBust reinforcement Learning with Adversarial Herds (ROLAH)

Input: m : size of the adversarial herd; k : the number of the worst adversaries to use; λ_p : step size for updating the agent policy; λ_a : step size for updating the adversary herd;

Output: $\hat{\theta}$: parameter for the agent policy.

Randomly initialize θ and $\{\phi_i\}_{i=1}^m$
 $t \leftarrow 0, \theta^t \leftarrow \theta, \phi_i^t \leftarrow \phi_i \quad \forall i \in [m]$

for $t = 0 : T - 1$ **do**

 {Update the adversarial herd.}

for $i = 1 : m$ **do**

 Estimate $R(\theta^t, \phi_i^t)$ by rolling out the agent π_{θ^t} with the adversary $\pi_{\phi_i^t}$

end for

 Construct $I_{\theta, \hat{\Phi}, k}$ with the estimations.

$\phi_j^{t+1} \leftarrow \phi_j^t - \lambda_a \nabla_{\phi} R(\theta^t, \phi_j^t) \quad \forall j \in I_{\theta, \hat{\Phi}, k}$

 {Update the agent policy.}

for $i = 1 : m$ **do**

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end for

$\hat{\theta} \leftarrow \theta^T$

We can use *any* DRL algorithms to train agent & adversary

Adversarial Herd with Optimization Over *Worst-k* Adversaries

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for $i = 1 : m$ **do**

Estimate $R(\theta^t, \phi_i^{t+1})$ by rolling out the agent π_{θ^t} with the adversary $\pi_{\phi_i^{t+1}}$

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Construct $I_{\theta, \hat{\Phi}, k}$ with the estimations.

$\theta^{t+1} \leftarrow \theta^t - \lambda_p \sum_{j \in I_{\theta, \hat{\Phi}, k}} \nabla_{\theta} R(\theta^t, \phi_j^{t+1})$ **Train agent**

end for

$\hat{\theta} \leftarrow \theta^T$

In practice, we can ensure the adversaries are distinct enough during update.

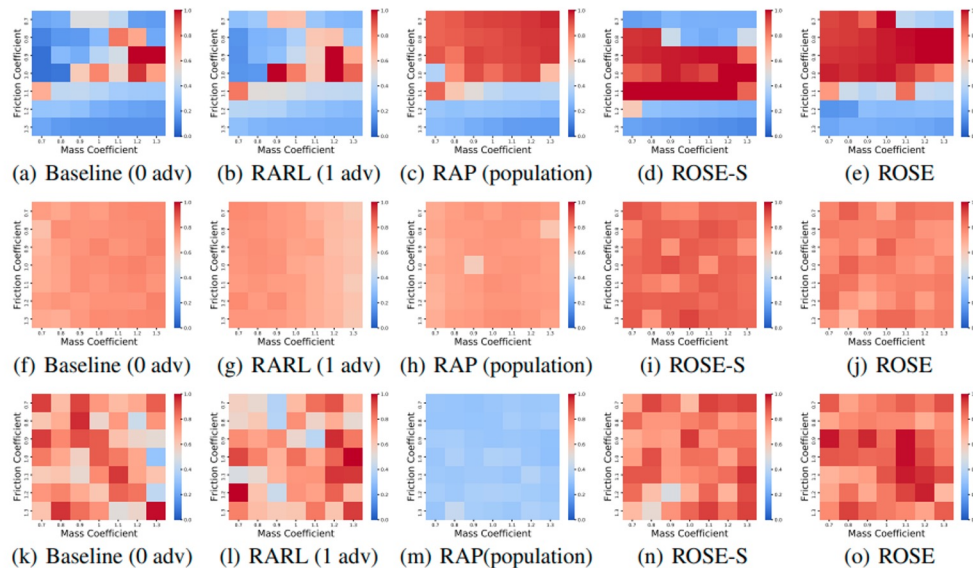
Evaluation on Standard Learning Benchmarks

1. Tasks: 5 MuJoCo environments in continuous action space
2. Core learning algorithms: TRPO (results in the slides), PPO, DDPG
3. Method comparison:
 - a. Baseline (e.g., TRPO itself w/o adversarial learning) [1]
 - b. RARL (1 adversary) [2]
 - c. RAP (population adversaires) [3]
 - d. M2TD3 (known uncertainty parameter set) [4]
 - e. ROSE (ours)

1. J. Schulman et al., "Trust region policy optimization", in *ICML 2015*
2. L. Pinto et al., "Robust Adversarial Reinforcement Learning", in *ICML 2017*
3. E. Vinitzky et al., "Robust reinforcement learning using adversarial populations", arXiv preprint arXiv:2008.01825, 2020
4. T. Tanabe et al., "Max-Min Off-Policy Actor-Critic Method Focusing on Worst-Case Robustness to Model Misspecification", in *NeurIPS, 2022*

Robustness to Test Conditions (Environmental Change)

1. Set both the friction and mass coefficients equal to 1.0 during training
2. Our method ROSE has competitive performance under varying test conditions
 - a. M2TD3 is not reported because it is already provided with the uncertainty parameter set for training.
 - b. Stein Variational Policy Gradient



Robustness to Agent Disturbance

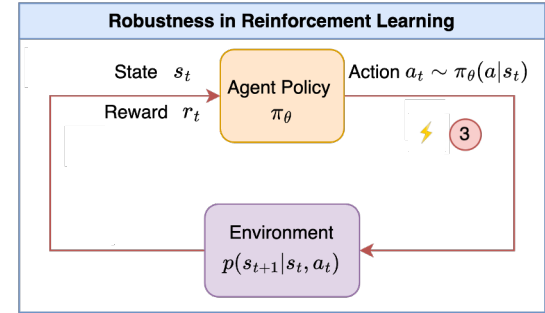
1. Overall, our method ROSE outperforms other methods.
2. M2TD3 is additionally provided with the uncertainty parameter set for training.
 - a. ROSE still outperforms M2TD3 in most scenarios with disturbances/adversarial attacks

Method	Baseline (0 adv)	RARL (1 adv)	RAP	M2TD3 (extra info)	ROSE-S (ours)	ROSE (ours)
Ant (No disturbance)	0.77±0.16	0.81±0.12	0.83±0.08	0.84±0.22	0.87±0.13	0.84±0.14
Ant (Action noise)	0.66±0.19	0.67±0.16	0.67±0.09	0.66±0.16	0.70±0.14	0.69±0.15
Ant (Adversary)	0.21±0.18	0.25±0.17	0.30±0.14	0.29±0.11	0.38±0.16	0.44±0.23
InvertedPendulum (No disturbance)	1.00±0	0.96±0.11	0.99±0.04	1.00±0	0.99±0.03	0.99±0.08
InvertedPendulum (Action noise)	0.91±0.13	0.91±0.15	0.95±0.10	0.97±0.16	0.96±0.13	0.96±0.11
InvertedPendulum (Adversary)	0.86±0.16	0.88±0.18	0.90±0.19	0.90±0.21	0.92±0.12	0.94±0.15
Hopper (No disturbance)	0.78±0.003	0.79±0.02	0.84±0	0.97±0.11	0.95±0.01	0.98±0.07
Hopper (Action noise)	0.71±0.001	0.74±0.004	0.80±0	0.77±0.07	0.91±0.006	0.87±0.01
Hopper (Adversary)	0.42±0.03	0.54±0.04	0.70±0.007	0.83±0.25	0.84±0.14	0.85±0.09
Half-Cheetah (No disturbance)	0.77±0.05	0.72±0.03	0.76±0.02	0.81±0.06	0.87±0.05	0.82±0.08
Half-Cheetah (Action noise)	0.59±0.2	0.76±0.04	0.67±0.1	0.68±0.13	0.76±0.16	0.73±0.13
Half-Cheetah (Adversary)	0.16±0.1	0.19±0.05	0.24±0.36	0.50±0.10	0.52±0.21	0.58±0.30
Walker2d (No disturbance)	0.85±0.27	0.84±0.43	0.43±0.02	0.88±0.31	0.84±0.44	0.86±0.38
Walker2d (Action noise)	0.78±0.31	0.80±0.28	0.36±0.04	0.79±0.21	0.83±0.37	0.84±0.23
Walker2d (Adversary)	0.36±0.26	0.34±0.12	0.34±0.22	0.21±0.43	0.68±0.23	0.70±0.17

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$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi)$$

$$\max_{\theta \in \Theta} \min_{\phi_1, \dots, \phi_m \in \Phi} \frac{1}{|I_{\theta, \hat{\Phi}, k}|} \sum_{i \in I_{\theta, \hat{\Phi}, k}} R(\theta, \phi_i)$$



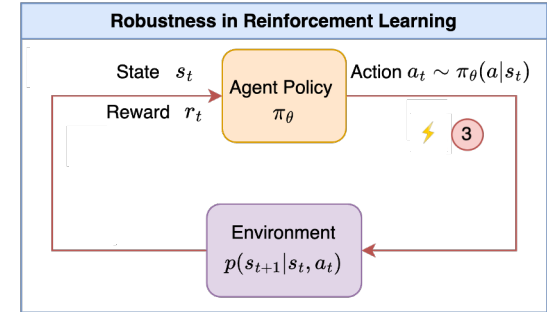
ROSE/RARL: Adversaries that incorporate domain knowledge

→ action space can be different between protagonist and adversary

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What if we do not have **any** domain knowledge for the action space?

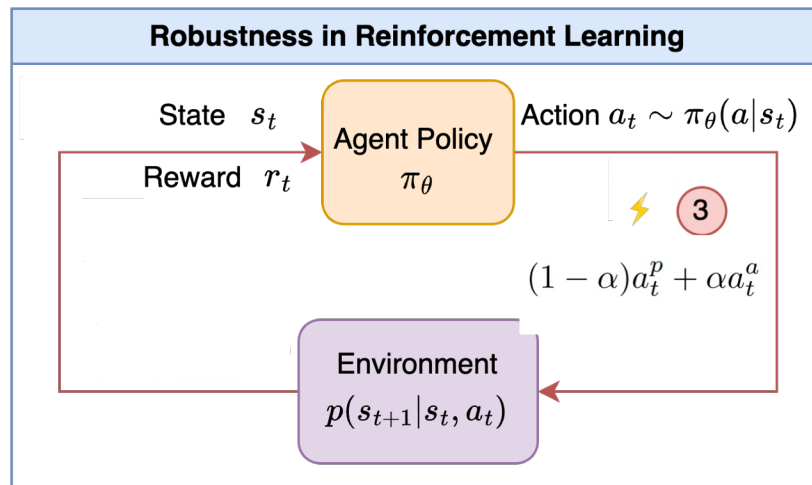
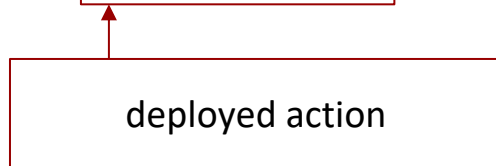
Noisy Action Robust MDPs (NR-MDPs)

$$R(\theta, \phi_i) \doteq \mathbb{E}_{s_0 \sim p_0} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, (1 - \alpha)a_t^p + \alpha a_t^a) \mid C \right], \text{ where } C = \{a_t^p \sim \pi_{\theta}, a_t^a \sim \pi_{\phi_i}\}$$

protagonist and adversary action

Noisy Action Robust MDPs (NR-MDPs)

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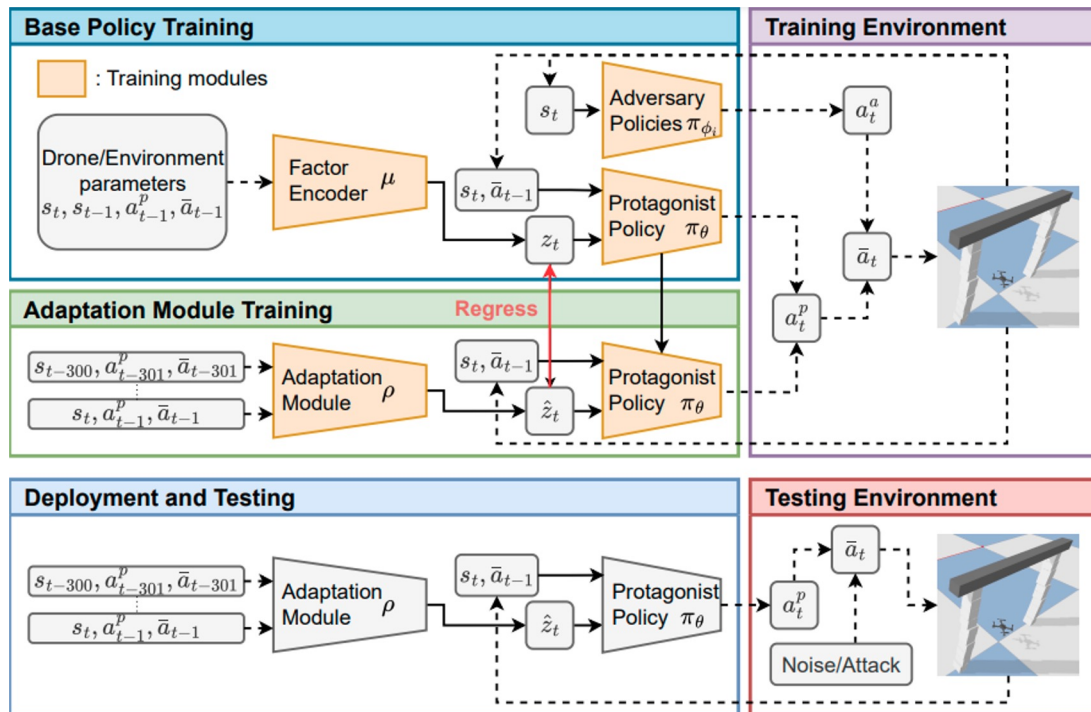


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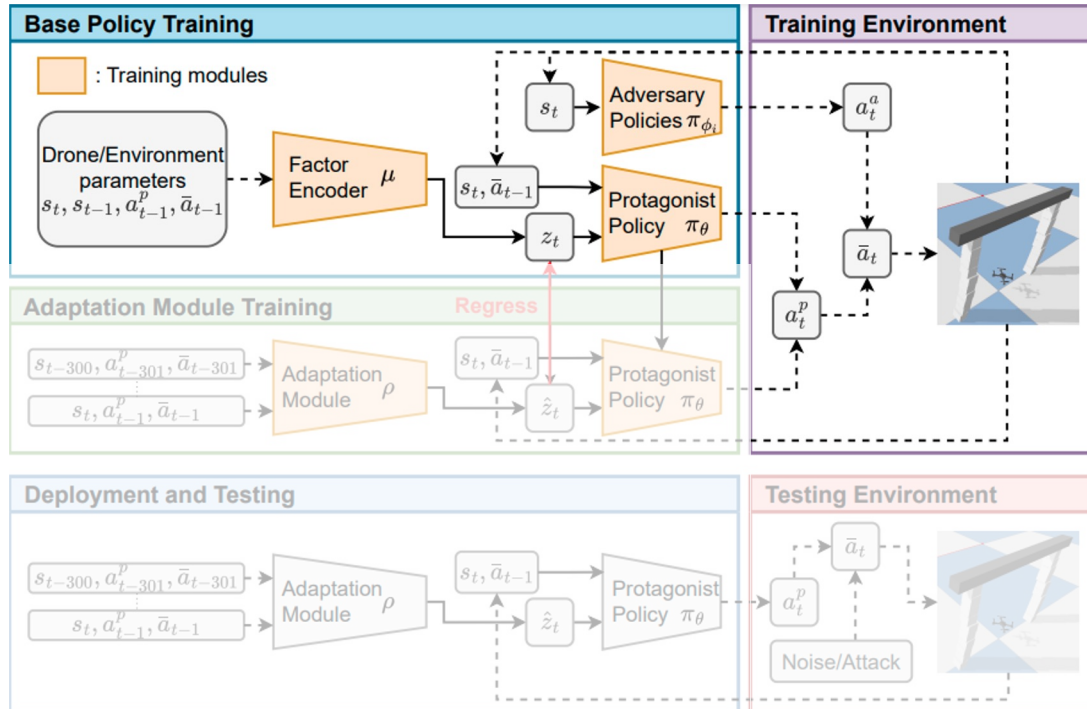
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REFORMA: Robust RL via Adaptive Adversary (ICRA24)



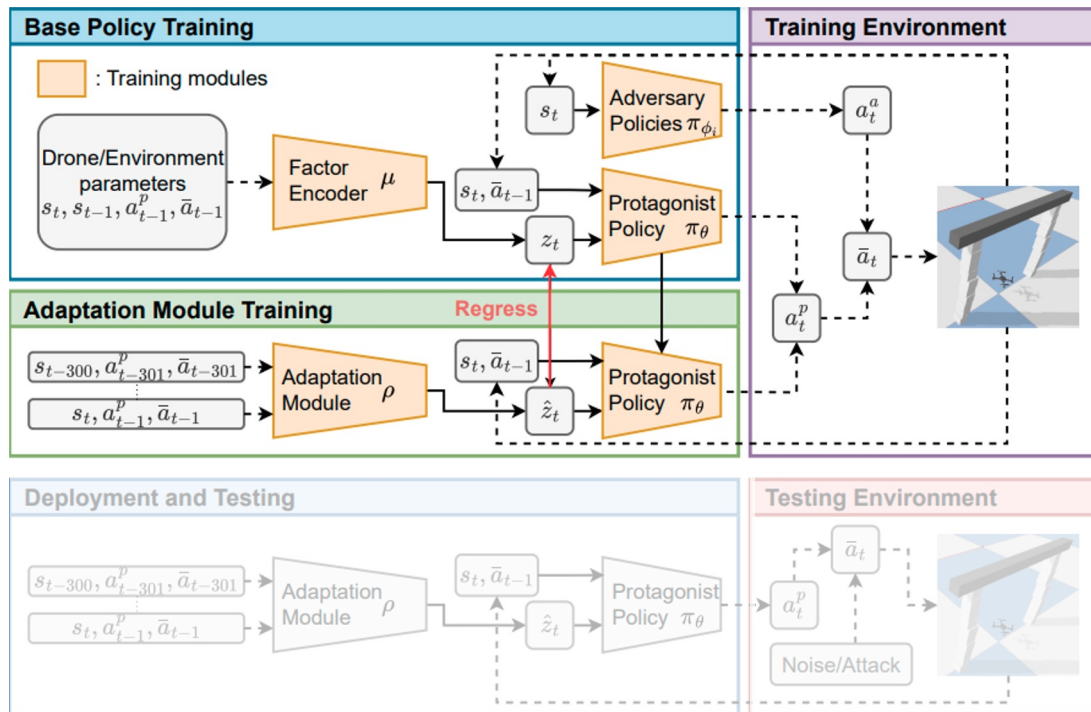
Problem: adversarial strength is unknown during evaluation

REFORMA: Robust RL via Adaptive Adversary (ICRA24)



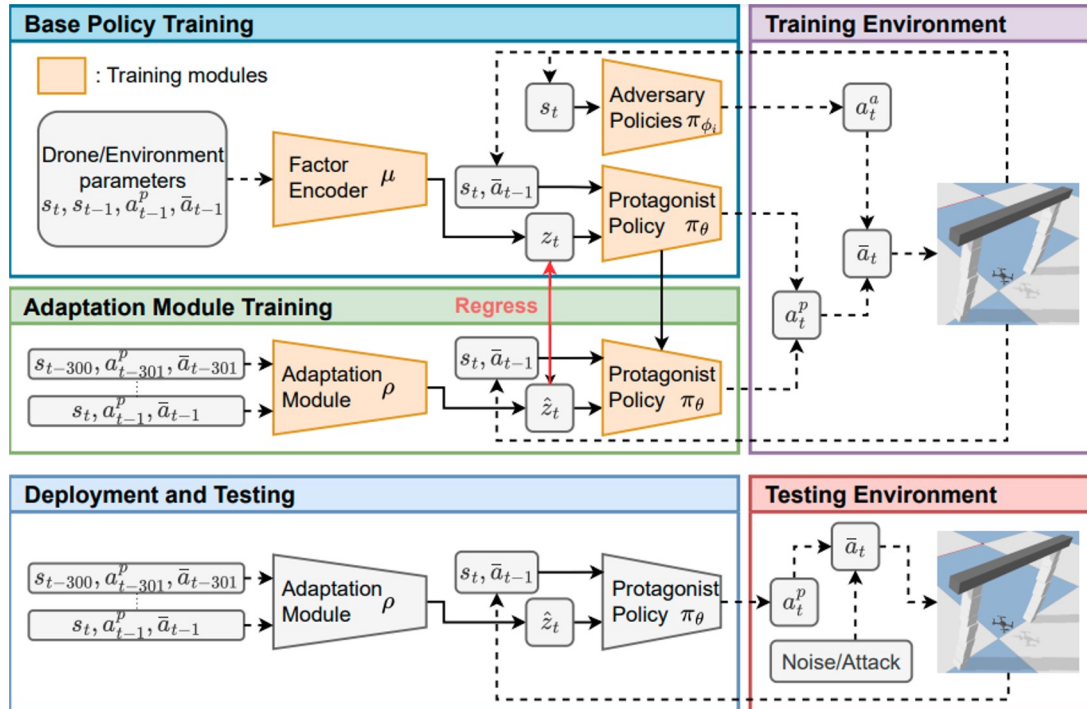
Base policy:
train protagonist and adversary policies, and factor encoder with attacked actions

REFORMA: Robust RL via Adaptive Adversary (ICRA24)



Adaptation module:
learn an adaptation module that takes state/actions history to capture drone and environment parameters.

REFORMA: Robust RL via Adaptive Adversary (ICRA24)



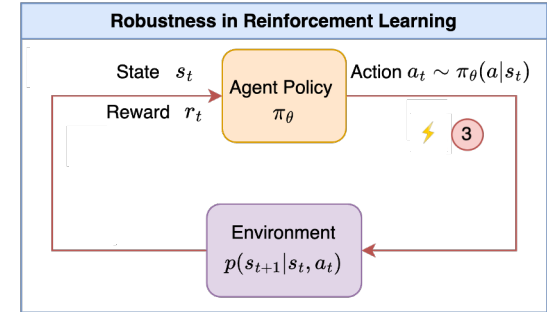
Deployment:

protagonist policy can be deployed with the inputs of the current state, previous attacked action and the latent space from adaptation module with unknown noise or attack.

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$$\max_{\theta \in \Theta} \min_{\phi \in \Phi} R(\theta, \phi)$$

$$\max_{\theta \in \Theta} \min_{\phi_1, \dots, \phi_m \in \Phi} \frac{1}{|I_{\theta, \hat{\Phi}, k}|} \sum_{i \in I_{\theta, \hat{\Phi}, k}} R(\theta, \phi_i)$$



If the action space is discrete, improvement due to the use a group of adversaries is not obvious.

[L4DC] We also studied the better exploration strategy under adversarial training.

- Propose robust RL via adversarial training with a group of adversaries
- Extend attackable actions in NR-MDP to adapt to a range of adversary strength
- Improve exploration under adversarial training for discrete action space using LMC

Thank you



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