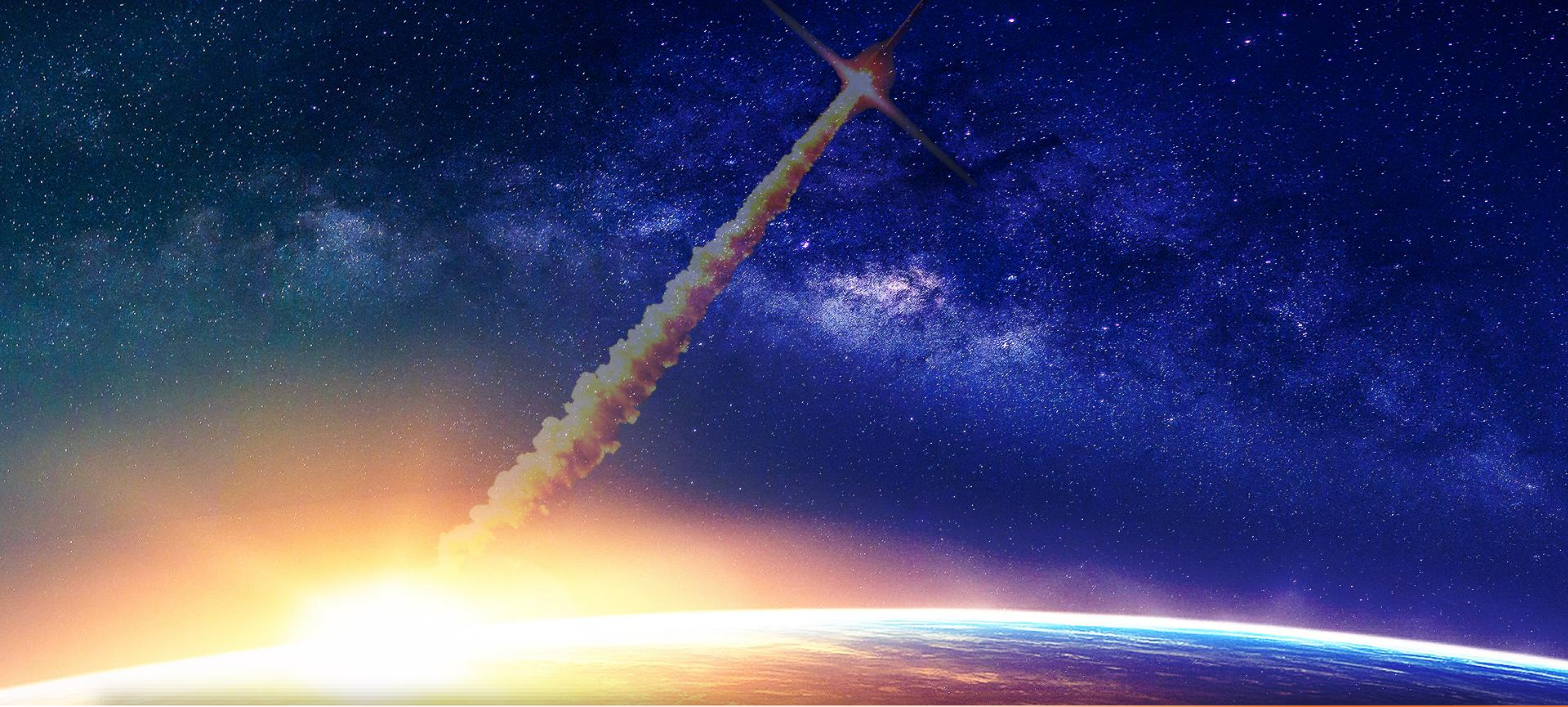


On The Topic of Time Sensitive Recursive Optimization of an Underactuated Docking Maneuver

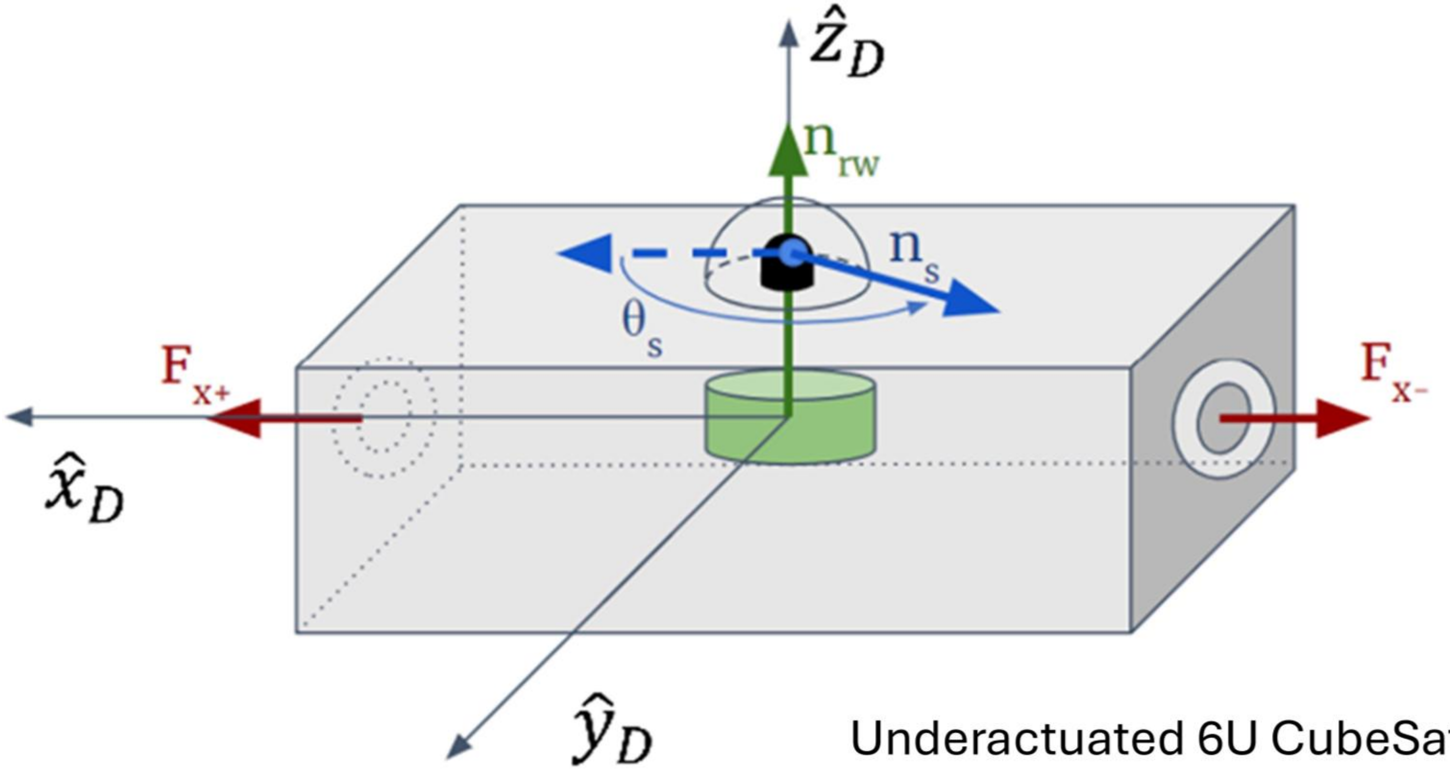
Anthony Aborizk, Bogachan Ondes, Norman Fitz-Coy

Motivation

- 36,000+ tracked debris items in Earth orbit (>10 cm)
- Estimated 1,000,000 untracked items (1 mm – 10 cm)
- Kessler Syndrome
- Active debris removal
    - Some techniques require docking for debris capture
    - We consider SmallSats for cost-effective operations
- Mission failures are a contributing factor to debris
- Increasing spacecraft launches → more potential failures*
- We present an autonomous nonlinear optimal control solution to an underactuated docking case study to
    - Improve cost efficiency by reducing the number of actuators
    - Provide options in the event of actuation failures

* Stephen A Jacklin. Small-Satellite Mission Failure Rates. Technical Report NASA/TM—2018–220034, NASA Ames Research Center, Moffett Field, CA, 2019.

Research Challenge Problem



Underactuated 6U CubeSat

*C. D. Petersen, S. Phillips, K. L. Hobbs, and K. Lang, “Challenge Problem: Assured Satellite Proximity Operations,” in *Proc. AAS/AIAA space flight mechanics meeting*, Virtual, 2021.

Research Challenge Problem

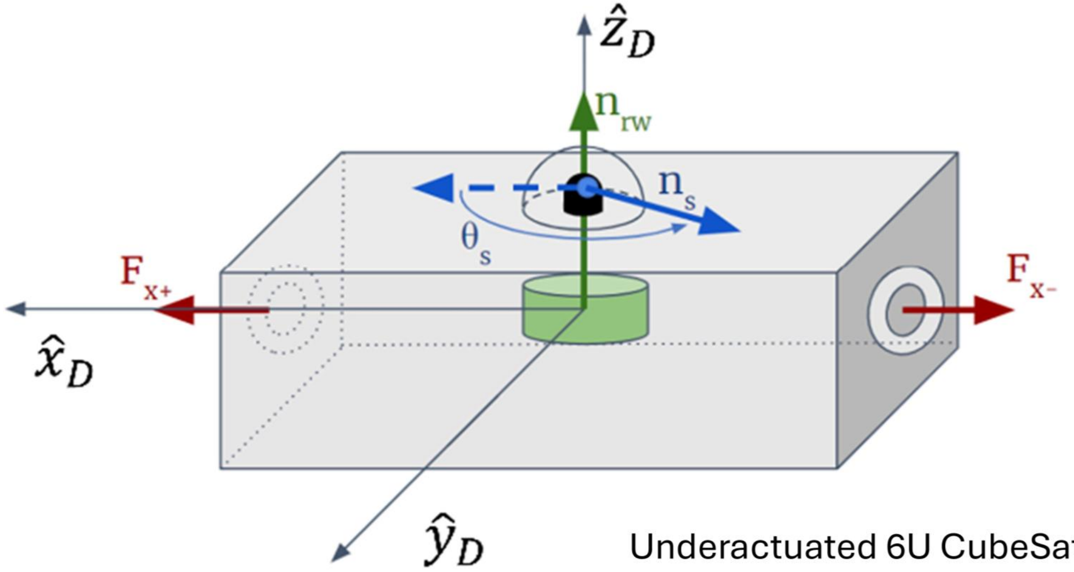
- The entire state space is defined as

$$\mathbf{s} \coloneqq \begin{pmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

- Let the control-input be defined as

$$\boldsymbol{u} = \begin{pmatrix} F_{\mathrm{x}} \\ \ddot{\psi}_z \end{pmatrix}$$

$$\dot{\mathbf{s}} = \mathbf{A}\mathbf{s} + \mathbf{B}(\theta)\boldsymbol{u}$$



Underactuated 6U CubeSat

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- The entire state space is defined as

$$\mathbf{s} := \begin{pmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

- Let the control-input be defined as

$$\boldsymbol{u} = \begin{pmatrix} F_{\mathrm{x}} \\ \ddot{\psi}_z \end{pmatrix}$$

$$\dot{\boldsymbol{s}} = \mathbf{A}\mathbf{s} + \mathbf{B}(\theta)\boldsymbol{u}$$

- Reaction wheel actuation is used to calculated angular acceleration using

$$\ddot{\theta}_z = -\frac{D_z \ddot{\psi}_z}{I_z}$$

- Coupling occurs in the control

$$F_{xy}^{\mathcal{O}} = \begin{bmatrix} cos(\theta) \\ sin(\theta) \end{bmatrix} F_x^{\mathcal{D}}$$

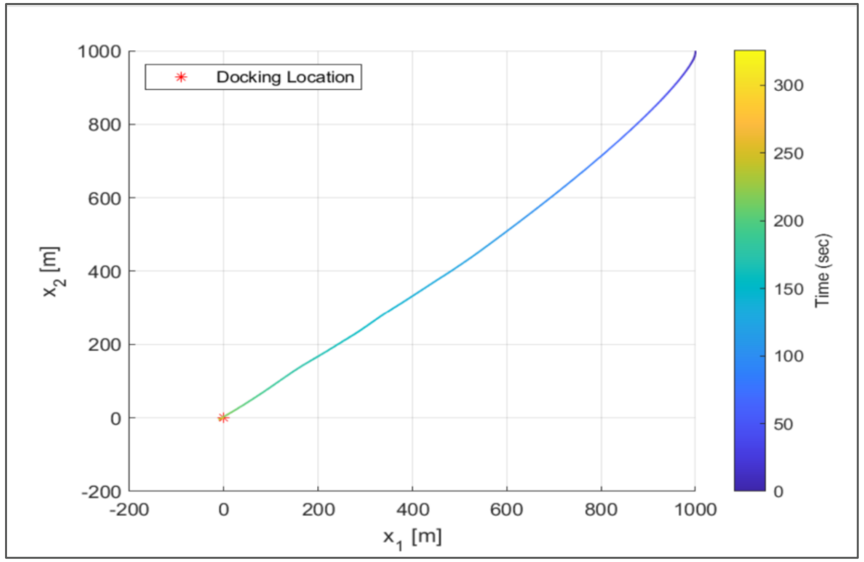
$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\eta^2 & 0 & 0 & 0 & 2\eta & 0 \\ 0 & 0 & 0 & -2\eta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

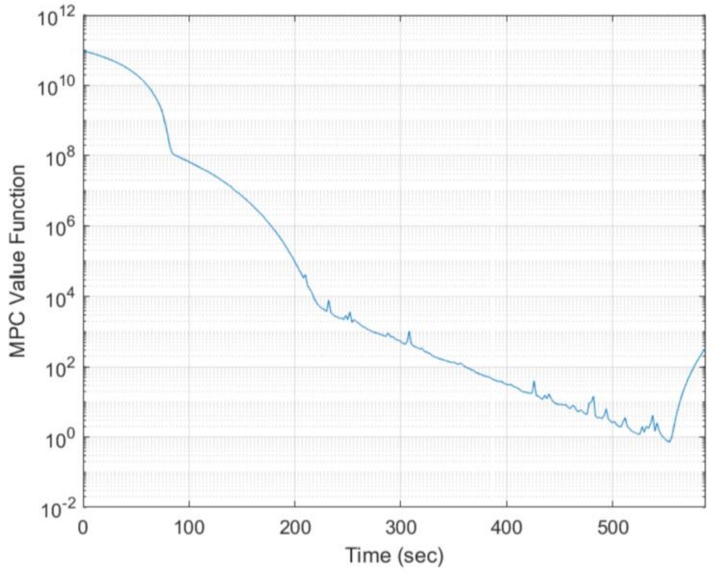
$$\boldsymbol{B}(\theta) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\cos(\theta)}{m_c} & 0 \\ \frac{\sin(\theta)}{m_c} & 0 \\ 0 & \frac{-D}{I_z} \end{pmatrix}$$

*C. D. Petersen, S. Phillips, K. L. Hobbs, and K. Lang, "Challenge Problem: Assured Satellite Proximity Operations," in *Proc. AAS/AIAA space flight mechanics meeting*, Virtual, 2021.

The Problem Cannot Be Solved with Classical Control

- Lie algebra tells us*:
  - When linearized, the system is neither controllable nor stabilizable
  - It cannot be transformed into a controllable linear system
    - Even with state feedback
  - The system is naturally unstable when no control is applied
- These limitations mean that:
  - Standard linear control methods won't work
  - Linear Lyapunov-based control design isn't possible
  - **Smooth (continuous) state feedback control won't stabilize the system**



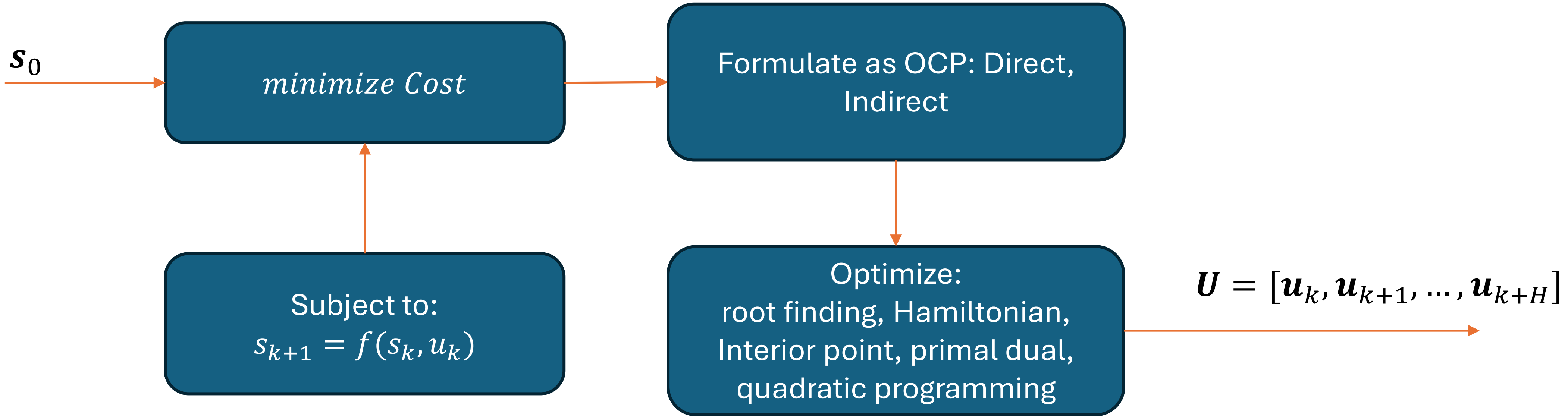


NMPC solution**

*A. A. Soderlund and S. Phillips, “Hybrid Systems Approach to Autonomous Rendezvous and Docking of an Underactuated Satellite,” *Journal of Guidance, Control, and Dynamics*, vol. 46, no. 10, pp. 1901–1918, Oct. 2023

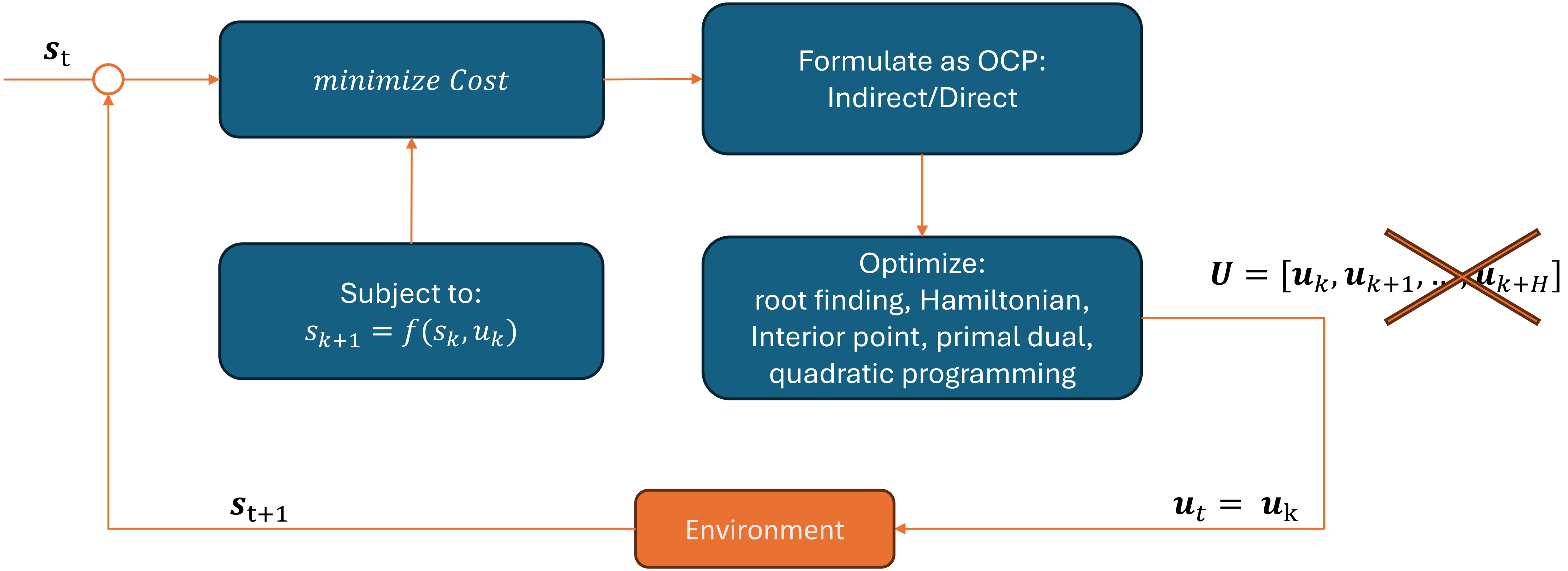
**A. Zaman, A. A. Soderlund, C. Petersen, and S. Phillips, “Autonomous Satellite Rendezvous and Proximity Operations via Model Predictive Control Methods,” Big Sky, Virtual, 2021

Take a Generic Optimal Control Problem (OCP)



* D. Garg, M. A. Patterson, C. Francolin, C. L. Darby, G. T. Huntington, W. W. Hager, and A. V. Rao, "Direct trajectory optimization and costate estimation of finite-horizon and infinite-horizon optimal control problems using a Radau pseudospectral method,"Computational Optimization and Applications, vol. 49, no. 2, pp. 335–358, Jun. 2011.[Online]. Available: http://link.springer.com/10.1007/s10589-009-9291-0

Add Recursion



*L. Gr¨une and J. Pannek, *Nonlinear model predictive control: theory and algorithms*, second edition ed., ser. Communications and control engineering. Cham, Switzerland: Springer, 2017

Stability of nMPC with Terminal Constraints

- Previous research identified concerns with stability
- We were able to generate trajectories with monotonicity
- However, they were sensitive to initializations
- Weakens requirement that the Lyapunov function must be continuous in the state
- We can enforce stability by including a terminal state constraint

$$V_H = \underset{\boldsymbol{s},\boldsymbol{u}}{\text{minimize}} \quad J_H\big(\boldsymbol{s}_0, \boldsymbol{u}(*)\big) := \sum_{k=0}^{H-1} \boldsymbol{s}_k^T Q \boldsymbol{s}_k + \boldsymbol{u}_k^T R \boldsymbol{u}_k$$

$$S.T. \quad \boldsymbol{s}_{k+1} = f(\boldsymbol{s}_k, \boldsymbol{u}_k)$$

$$\boldsymbol{u} \in \mathcal{U}$$

$$\boldsymbol{s} \in \mathcal{S}$$

$$\boldsymbol{s}_{k+H-1} = \boldsymbol{0}_{\boldsymbol{6}}$$

Theorem 1*: For a discrete-time MPC problem with a terminal state condition $\boldsymbol{s}_{\boldsymbol{N}} = 0$, and if V_N is continuous at $\boldsymbol{s}_{\boldsymbol{0}} = 0$ and L satisfies Requirements 1 and 2, then the origin is an asymptotically stable equilibrium of the discrete-time system.

Requirement 1: $L(0,0) = 0$

Requirement 2: There exists a non-decreasing function $\gamma: [0, \infty) \to [0, \infty)$ such that $\gamma(0) = 0$ and $0 < \gamma\big(\|\boldsymbol{s}, \boldsymbol{u}\|\big) \leq L(\boldsymbol{s}, \boldsymbol{u})$ for all $(\boldsymbol{s}, \boldsymbol{u}) \neq 0$, in which $\|*,*\|$ is a norm on the pair $(\boldsymbol{s}, \boldsymbol{u})$

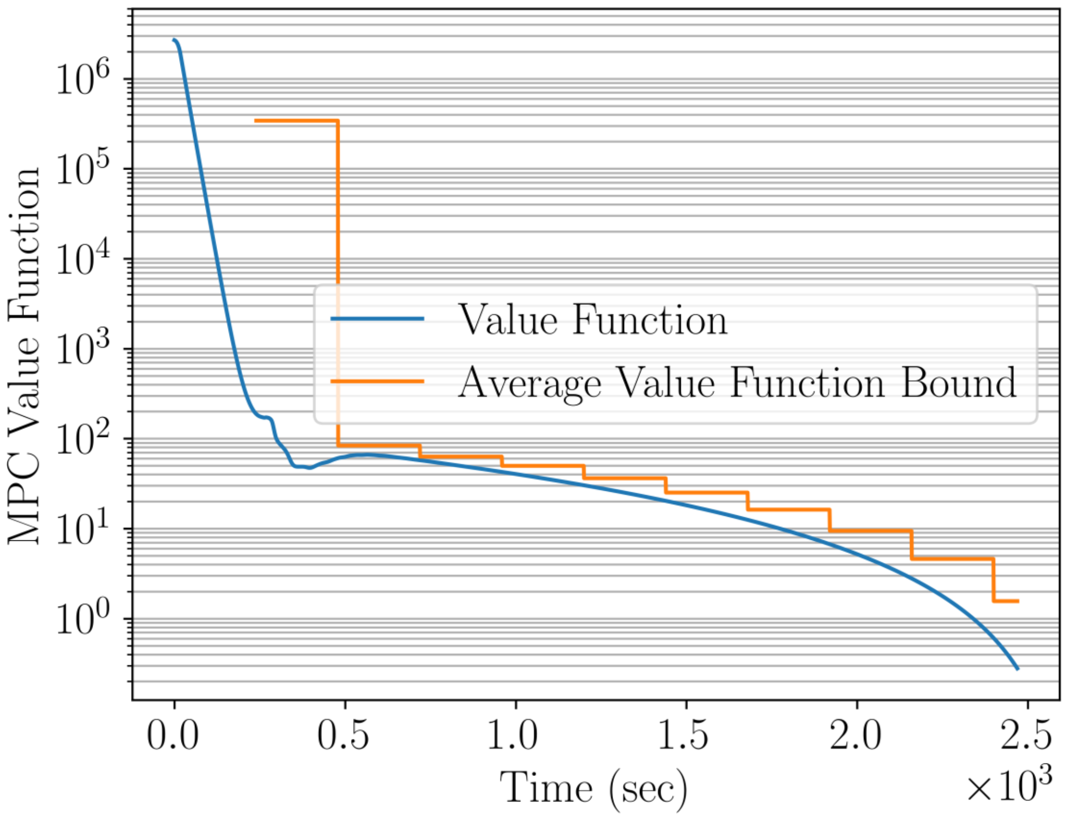
*E. S. Meadows, M. A. Henson, J. W. Eaton, and J. B. Rawlings, "Receding horizon control and discontinuous state feedback stabilization," *International Journal of Control*, vol. 62, no. 5, pp. 1217–1229, Nov. 1995, publisher: Taylor & Francis

**H. H. Sohrab, *Basic Real Analysis*, 2nd ed., ser. SpringerLink B¨ucher. New York, NY: Birkhauser, 2014

NMPC without terminal constraints

- Adding a terminal state does a lot
  - Guarantees asymptotic stability
  - Explicit handling of constraints
  - Fills the gap in understanding in the literature
- However
  - Requires a long prediction horizon
    - Larger problem
    - Longer computation time
- Can we solve this without a terminal state?
  - Linear MPC rules don't apply
  - Can't induce monotonicity
- Use the average cost to bound the value function

$$\boxed{V_H \leq \frac{1}{N}\sum_{n=0}^{N-1} V_{H_n}}$$



Benchmark Simulation Setup

$$V_H = \underset{\boldsymbol{s},\boldsymbol{u}}{\text{minimize}} \quad J_H(\boldsymbol{s}_0, \boldsymbol{u}(*)) := \sum_{k=0}^{H-1} \boldsymbol{s}_k^T Q \boldsymbol{s}_k + \boldsymbol{u}_k^T R \boldsymbol{u}_k$$

$$S.T. \quad \boldsymbol{s}_{k+1} = f(\boldsymbol{s}_k, \boldsymbol{u}_k)$$

$$|F_x| \leq 2\ N$$

$$|\ddot{\psi}_z| \leq 1604.28\ \frac{deg}{s^2}$$

$$|x|, |y| \leq 5{,}000\ m$$

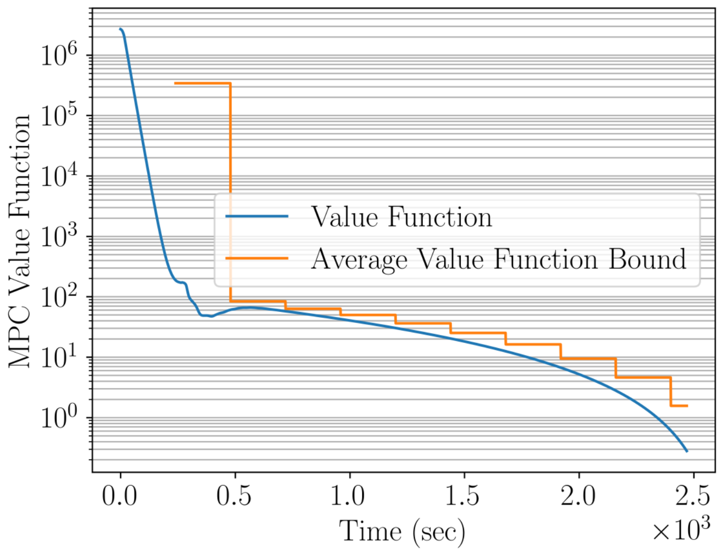
$$|\dot{x}|, |\dot{y}| \leq 10\ \frac{m}{s}$$

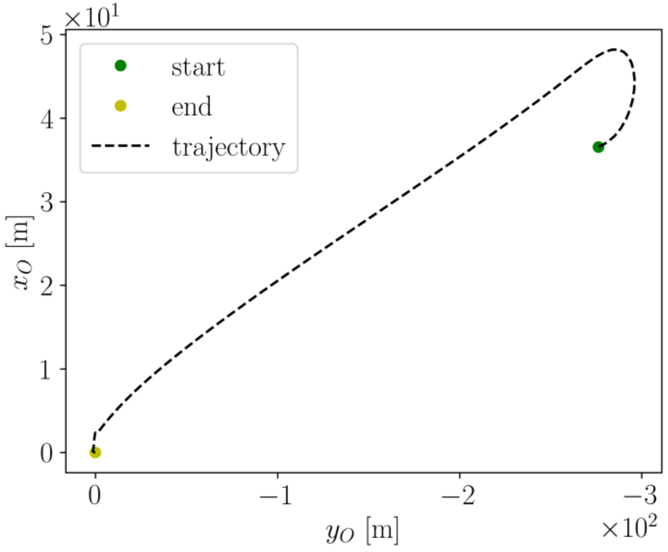
$$|\theta| \leq 180\ deg$$

$$|\dot{\theta}| \leq 2\ \frac{deg}{s}$$

$$|\ddot{\theta}| \leq 1\ \frac{deg}{s^2}$$

$$\cancel{\boldsymbol{s}_{k+H-1} = \boldsymbol{0}_6}$$





Benchmark Simulation Setup

$$V_H = \underset{s,u}{\text{minimize}} \quad J_H(\boldsymbol{s}_0, \boldsymbol{u}(*)) \coloneqq \sum_{k=0}^{H-1} \boldsymbol{s}_k^T Q \boldsymbol{s}_k + \boldsymbol{u}_k^T R \boldsymbol{u}_k$$

$$S.T. \quad \boldsymbol{s}_{k+1} = f(\boldsymbol{s}_k, \boldsymbol{u}_k)$$

$$\left.\begin{aligned} |F_x| &\le 2\ N \\ |\ddot{\psi}_z| &\le 1604.28\ \frac{deg}{s^2} \end{aligned}\right\} \text{Actuation Constraints}$$

$$|x|, |y| \le 5{,}000\ m$$

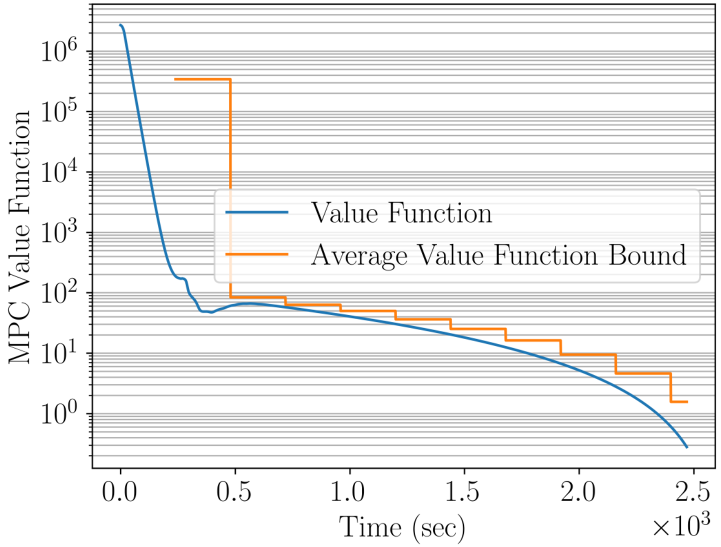
$$|\dot{x}|, |\dot{y}| \le 10\,\frac{m}{s}$$

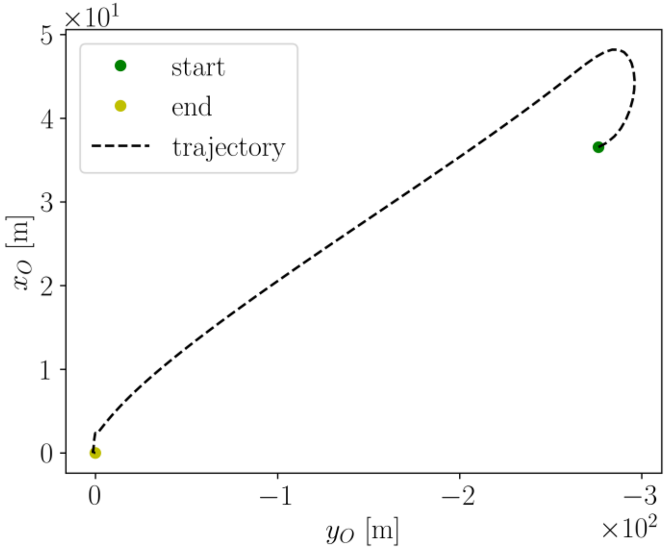
$$|\theta| \le 180\ deg$$

$$|\dot{\theta}| \le 2\ \frac{deg}{s}$$

$$|\ddot{\theta}| \le 1\ \frac{deg}{s^2}$$

$$\cancel{\boldsymbol{s}_{k+H-1} = \boldsymbol{0}_6}$$





Benchmark Simulation Setup

$$V_H = \underset{\boldsymbol{s},\boldsymbol{u}}{\text{minimize}} \quad J_H(\boldsymbol{s}_0, \boldsymbol{u}(*)) \coloneqq \sum_{k=0}^{H-1} \boldsymbol{s}_k^T Q \boldsymbol{s}_k + \boldsymbol{u}_k^T R \boldsymbol{u}_k$$

$$S.T. \quad \boldsymbol{s}_{k+1} = f(\boldsymbol{s}_k, \boldsymbol{u}_k)$$

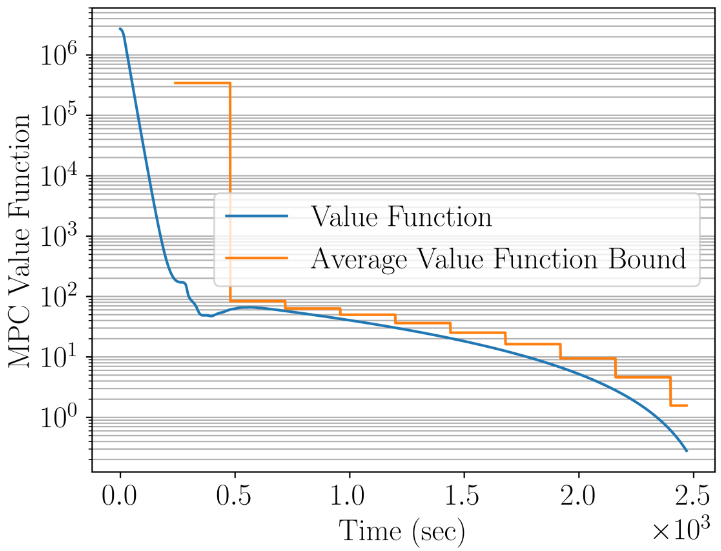
$$|F_x| \leq 2\ N$$

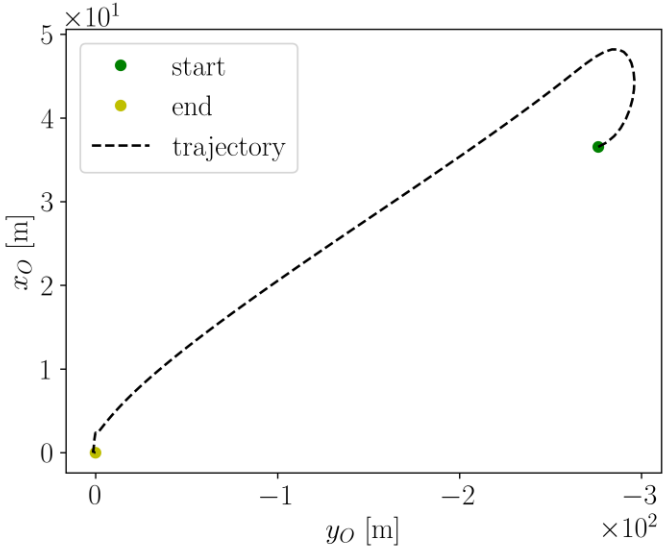
$$|\ddot{\psi}_z| \leq 1604.28\ \frac{deg}{s^2}$$

$$\left.\begin{array}{l} |x|, |y| \leq 5{,}000\ m \\ |\dot{x}|, |\dot{y}| \leq 10\ \frac{m}{s} \\ |\theta| \leq 180\ deg \\ |\dot{\theta}| \leq 2\ \frac{deg}{s} \end{array}\right\} \text{State Constraints}$$

$$|\ddot{\theta}| \leq 1\ \frac{deg}{s^2}$$

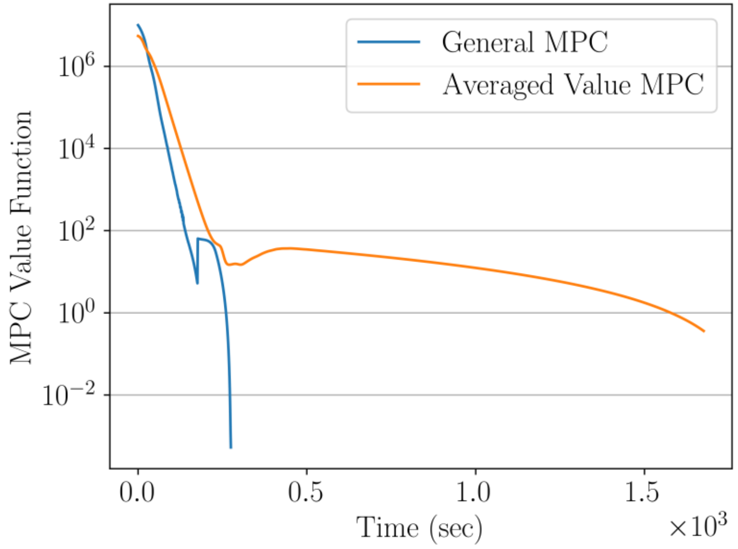
$$\cancel{\boldsymbol{s}_{k+H-1} = \boldsymbol{0}_6}$$

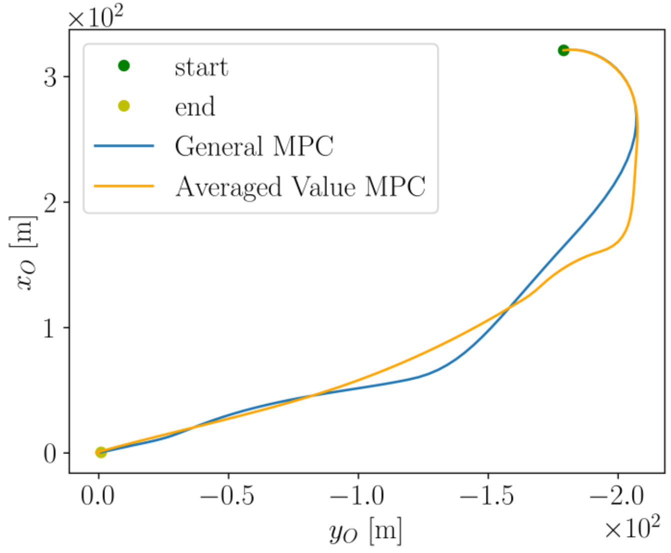




Comparison

- General vs averaged value MPC approaches
- Similar looking trajectories
- General MPC
  - Consistently more optimal
  - Drives the deputy to the docking state in fewer timesteps
  - Potential discontinuity captured
  - Takes 17,569 seconds to complete in simulation
  - Average 9.34 seconds of computation per mission timestep (2 seconds allowable)
- Averaged value MPC
  - Longer mission time
  - Takes 971 seconds to complete in simulation
  - Average 0.58 seconds of computation per mission timestep
  - **16-fold reduction in computational time**





What happens if we make the constraints more realistic?

$$\underset{s,u}{\text{minimize}} \quad J_H(\boldsymbol{s}_0, \boldsymbol{u}(*)) \coloneqq \sum_{k=0}^{H-1} \boldsymbol{s}_k^T Q \boldsymbol{s}_k + \boldsymbol{u}_k^T R \boldsymbol{u}_k$$

$$S.T. \quad \boldsymbol{s}_{k+1} = f(\boldsymbol{s}_k, \boldsymbol{u}_k)$$

$$|F_x| \leq \cancel{2} \rightarrow 0.1\ N$$

$$|\ddot{\psi}_z| \leq \cancel{1604.28} \rightarrow 15 \frac{deg}{s^2}$$

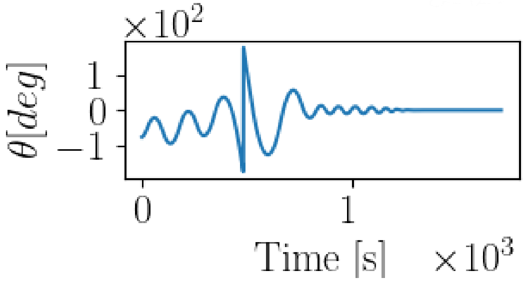
$$|x|, |y| \leq 5{,}000\ m$$

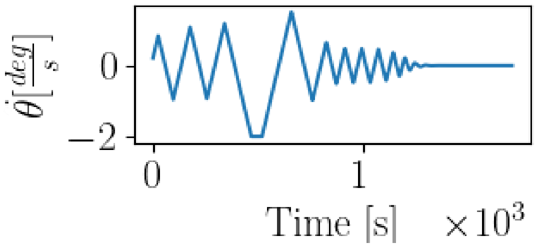
$$|\dot{x}|, |\dot{y}| \leq \cancel{10} \rightarrow 2 \frac{m}{s}$$

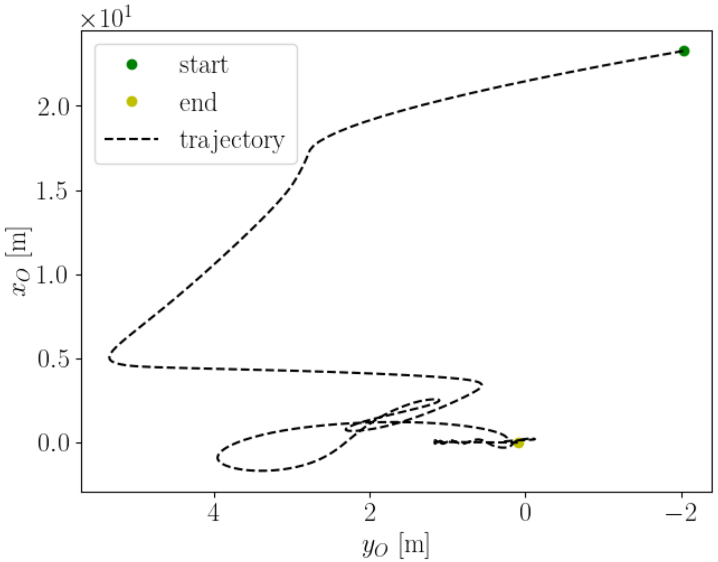
$$|\theta| \leq 180 \text{ deg}$$

$$|\dot{\theta}| \leq 2 \frac{deg}{s}$$

$$|\ddot{\theta}| \leq 1 \frac{deg}{s^2}$$







Add Damping to the Orientation

- The orientation equations,

$$\ddot{\theta}_z = -\frac{D_z\ddot{\psi}_z}{I_z},$$

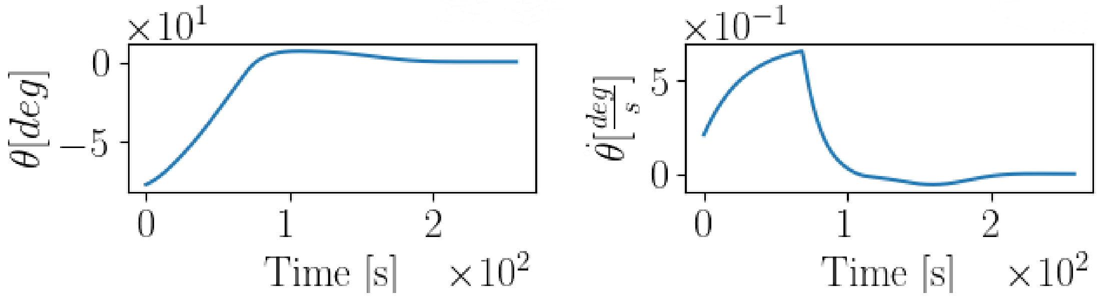
- become

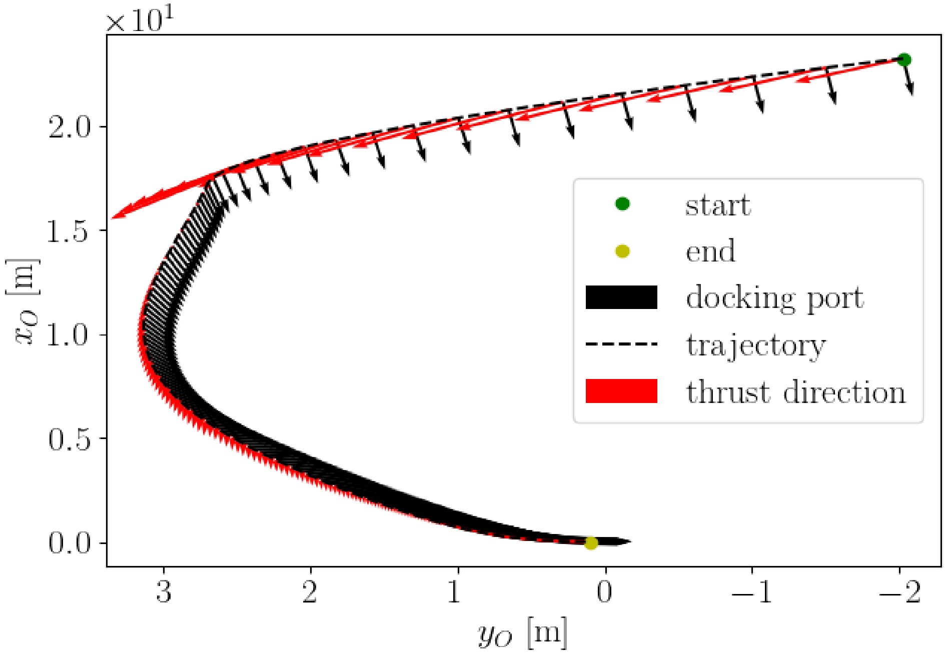
$$\ddot{\theta}_z = -\frac{D_z\ddot{\psi}_z}{I_z} - \frac{c}{I_z}\dot{\theta} + \frac{kB}{I_z}u_m$$

- c $=$ internal friction/hysteresis loss
- kB $=$ magnetorquer torque constant
- u_m $=$ the control effort

$$kBu_m = -kBK\dot{\theta}$$

$$\boxed{\ddot{\theta}_z = -\frac{D_z\ddot{\psi}_z}{I_z} - \frac{c}{I_z}\dot{\theta}}$$



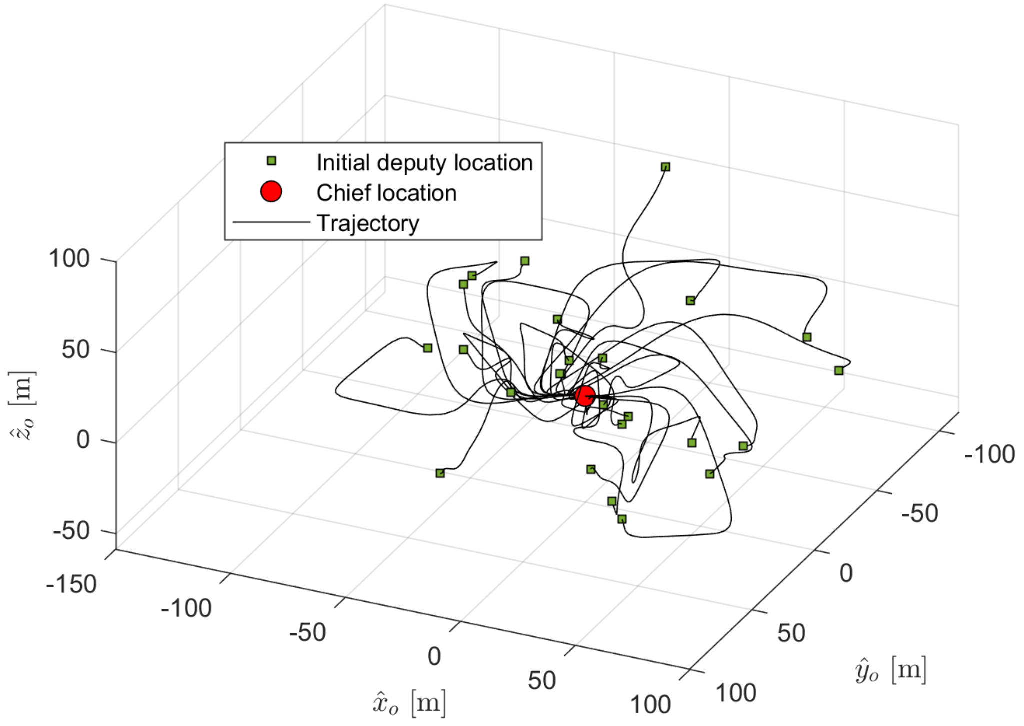


Future Work

- Elliptical orbits

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}-k\omega^{\frac{3}{2}}x + 2\omega\dot{z} + \dot{\omega}z + \omega^2 x\\ -k\omega^{\frac{3}{2}}y\\ 2k\omega^{\frac{3}{2}}z - 2\omega\dot{x} - \dot{\omega}x + \omega^2 z\end{bmatrix} + \boldsymbol{a}_f$$

- Explore additional constraints
  - Keep deputy within the line of sight of the chief
  - Sun exclusion angles
  - Safe modes
  - Varying reference/uncooperative chief
- Expand this to a real-time solutions
  - non-linear model predictive control
  - model-based reinforcement learning
    - With a3c
  - Imitation learning



Questions

3 Ingredients for Stability with No Terminal Cost or Constraints

1. Is the optimal stage cost bounded above and below by suitable Kappa infinity functions?

$$\alpha_1(|\mathbf{s}-\mathbf{s}^*|) \leq \mathrm{l}^*(\mathbf{s}) \leq \alpha_2(|\mathbf{s}-\mathbf{s}^*|)$$

   - for suitable $\alpha_1, \alpha_2 \in \kappa_\infty$

2. The value function is upper bounded by some asymptotic/exponential function

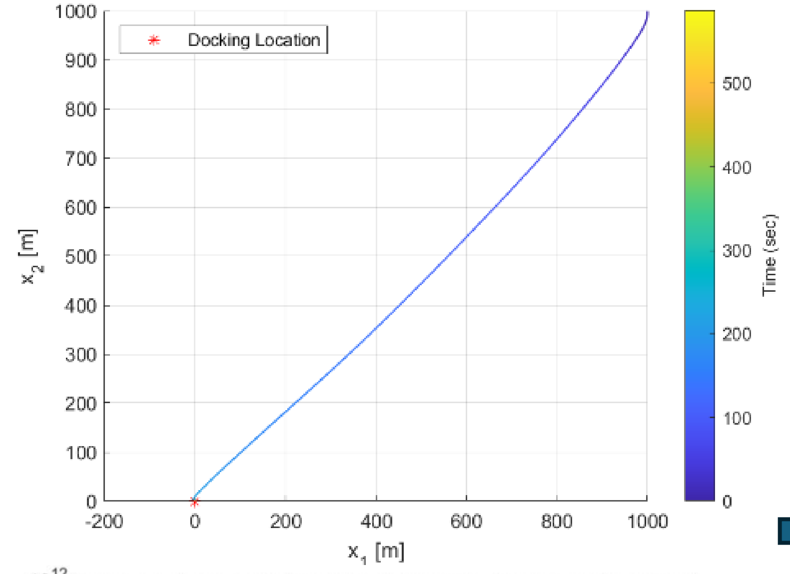
$$l\big(s_u(n,\boldsymbol{s}), u(n)\big) \leq \beta(l^*(\boldsymbol{s}), n)$$

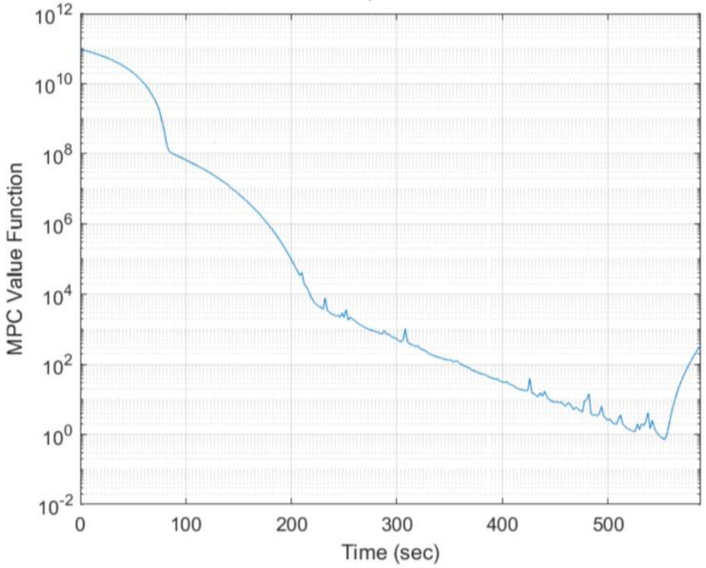
   - we can't solve this explicitly
   - But we can use the requirement to tune our controller

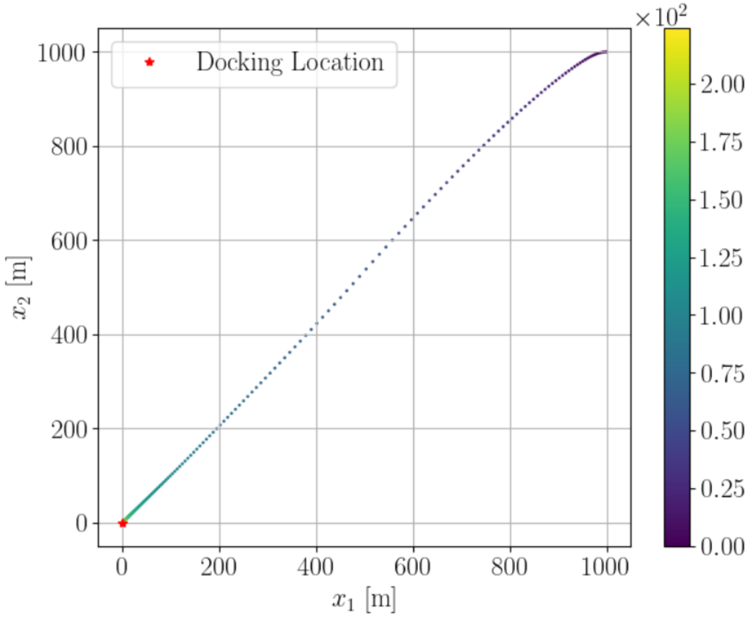
3. What level of optimality are we looking for?

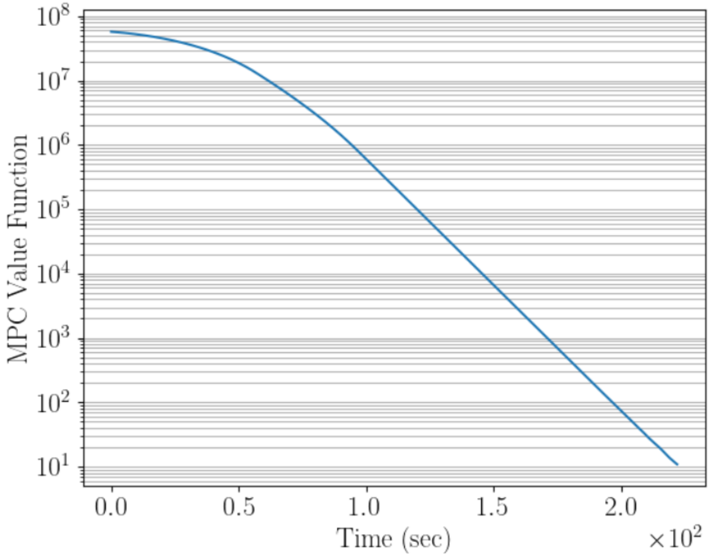
$$V_N(n,x) \ \geq \ \alpha\ell(n,x,\mu_N(n,x)) \ + \ V_N\Big(n \ + \ 1, f\big(x,\mu_N(n,x)\big)\Big)$$

   - for $\alpha \in (0,1]$







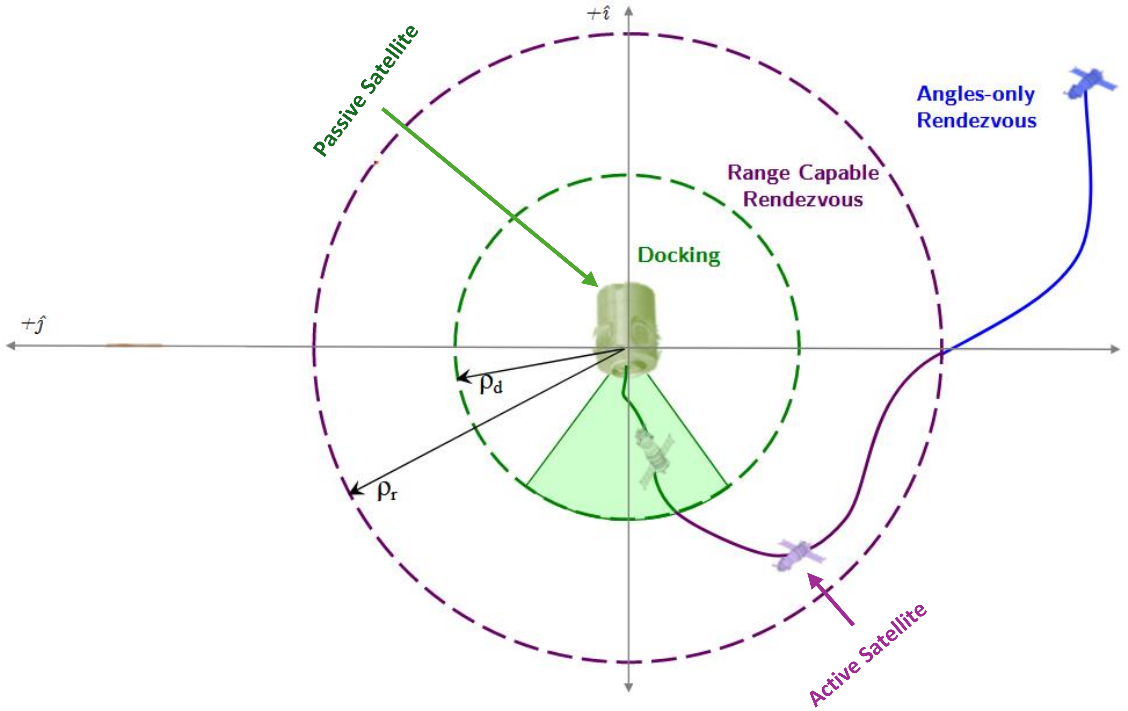


Stability Through Numerical Approaches

- Time to implement: 302 s
- Previous: Over night
- Overall, trajectory is completed faster
- Converges to 0.2 m box instead of 0.5 m box
- ~2-degree error

Autonomous Debris Capture

- Building guidance algorithms for capture/docking
- State of the art works well in static environments
    - Model Predictive Control
    - Artificial Potential Functions
    - Mixed Integer Linear Programming
- Technical challenges:
    - Nonlinear/discontinuous dynamics
    - Constrained actuation problems



Concept of operations for some ARPOD missions*

*C. Jewison and R. S. Erwin, "A spacecraft benchmark problem for hybrid control and estimation," in 2016 IEEE 55th Conference on Decision and Control (CDC), Dec. 2016, pp. 3300–3305.