Modular Adaptive Safety with a RISE-Based Disturbance Observer

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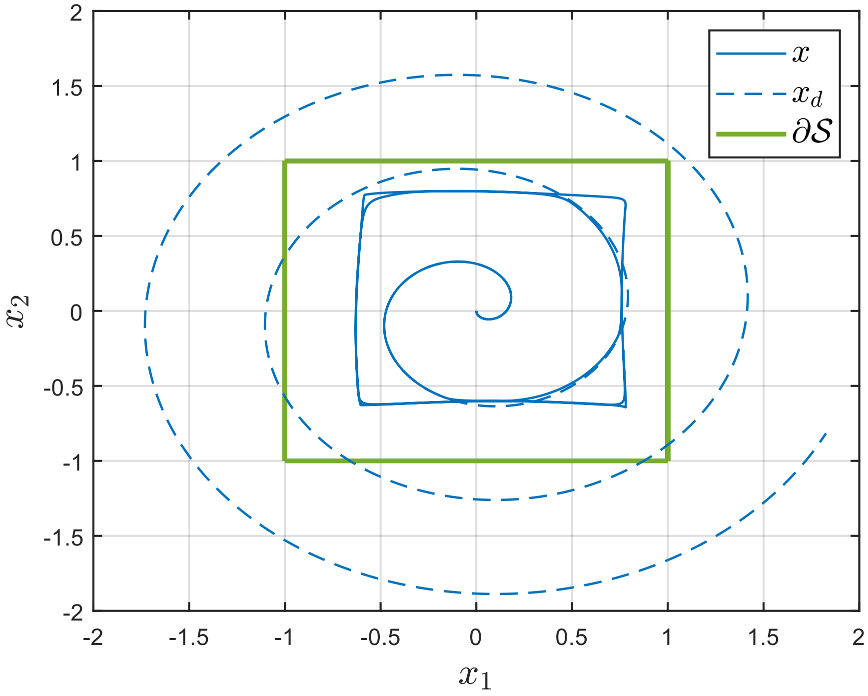
Safety

- Safety is conservative in the presence of uncertainty

- Must account for the worst case



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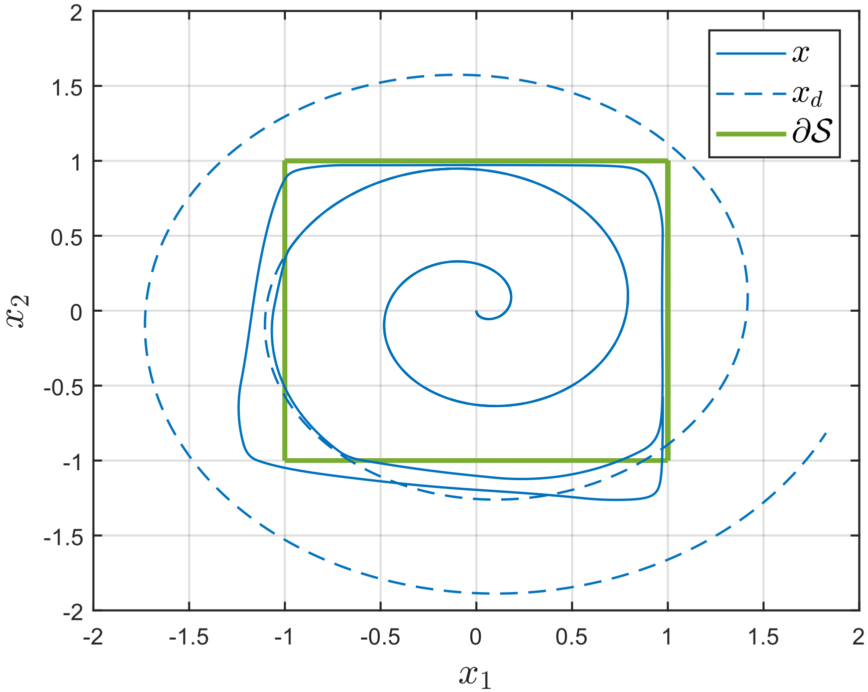
Adaptive Safety

- Can integrate feedforward estimates of dynamics such as deep neural networks or Gaussian processes

- These methods do not provide deterministic guarantees of identification



Control Objective

$$\dot{x} = f(x) + g(x)u + d(x) \quad u \in \Psi(x)$$

- $d\colon \mathbb{R}^n \to \mathbb{R}^n$ is unknown
- Design a controller so that

$$\mathcal{S} \triangleq \{x \in \mathbb{R}^n : B(x) \leq 0\}$$

is forward invariant, where

$$B(x) \triangleq [B_1(x), B_2(x), \ldots, B_d(x)]^T$$

- Safe set described by multiple continuously differentiable control barrier function candidates

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- For forward invariance we need

$$\Gamma_i\left(x,u\right) \triangleq \nabla B_i^T\left(x\right)\left(f\left(x\right)+g\left(x\right)u\right)+\nabla B_i^T\left(x\right)d\left(x\right) \leq -\gamma_i\left(x\right)$$

$$\kappa^*\left(x\right) \triangleq \arg\min_{u\in\mathbb{R}^m} Q\left(x,u\right)$$

$$\text{s.t. } \Gamma\left(x,u\right) \leq -\gamma\left(x\right)$$

Adaptive Safety

- For forward invariance we need

$$\Gamma_i(x,u) \triangleq \nabla B_i^T(x)\left(f(x)+g(x)u\right)+\nabla B_i^T(x)\,d(x) \leq -\gamma_i(x)$$

- Could use

$$\bar{\Gamma}_i(x,u) \triangleq \nabla B_i^T(x)\left(f(x)+g(x)u\right)+\chi_i(x) \leq -\gamma_i(x)$$

$$\chi_i(x) \triangleq \bar{d}\left\|\nabla B_i(x)\right\|$$

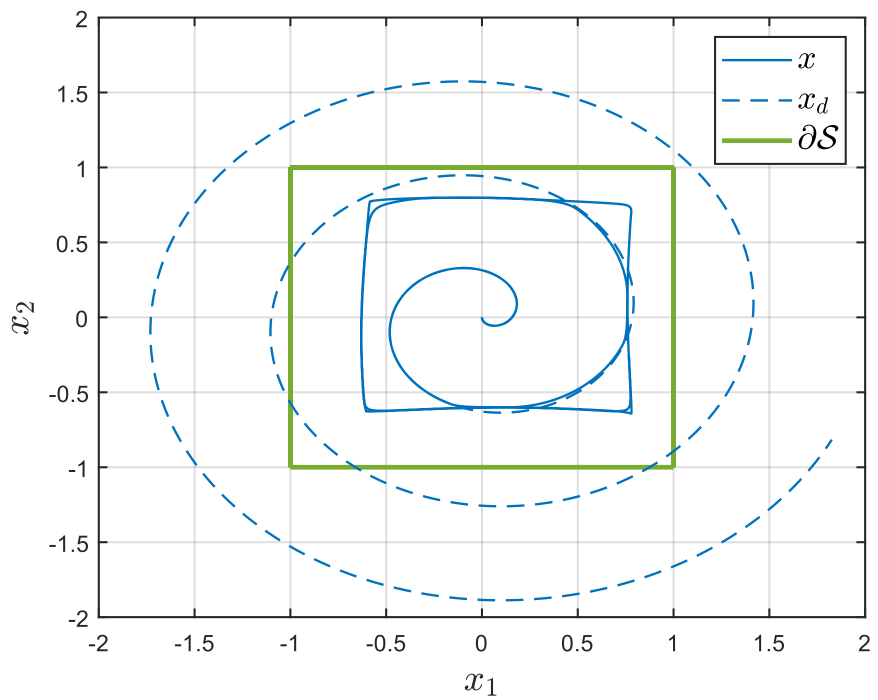
Adaptive Safety

- For forward invariance we need

$\Gamma_i(x,u) \triangleq$ ... $x) \leq -\gamma_i(x)$

- Could use

$\bar{\Gamma}_i(x,u$ ... $\leq -\gamma_i(x)$



Adaptive Safety

- For forward invariance we need

$$\Gamma_i\left(x,u\right) \triangleq \nabla B_i^T\left(x\right)\left(f\left(x\right)+g\left(x\right)u\right)+\nabla B_i^T\left(x\right)d\left(x\right) \leq -\gamma_i\left(x\right)$$

- Could use

$$\bar{\Gamma}_i\left(x,u\right) \triangleq \nabla B_i^T\left(x\right)\left(f\left(x\right)+g\left(x\right)u\right)+\chi_i\left(x\right) \leq -\gamma_i(x)$$

$$\chi_i\left(x\right) \triangleq \bar{d}\left\|\nabla B_i\left(x\right)\right\|$$

- But we'd rather have

$$\chi_i(x(t)) \searrow \nabla B_i\left(x(t)\right)^T d\left(x(t)\right)$$

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RISE-Based Disturbance Observer

$$\dot{\hat{x}} = f(x) + g(x)u + \hat{d} + \alpha\tilde{x}$$

$$\dot{\hat{d}} = k_d\left(\dot{\tilde{x}} + \alpha\tilde{x}\right) + \tilde{x} + \beta \text{dir}(\tilde{x})$$

$$\tilde{x} = x - \hat{x}$$

$$\text{dir}(\tilde{x}) \triangleq \begin{cases} \tilde{x}/\|\tilde{x}\| & \tilde{x} \neq 0 \\ 0 & o/w \end{cases}$$

- We show that

$$\left\|\tilde{d}(t)\right\| \leq \left\|\tilde{d}(0)\right\| e^{-\lambda t}$$

- Provided

$$\left\|\dot{d}(x(t))\right\| \leq c_1$$

$$\left\|\ddot{d}(x(t))\right\| \leq c_2$$

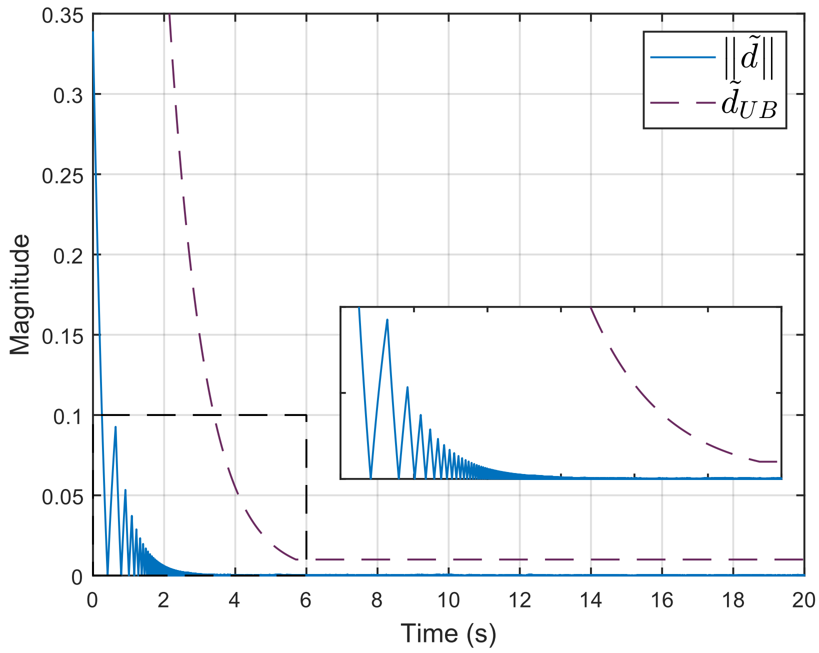
and

$$\beta > c_1 + \frac{c_2}{\max(1, \alpha - k_d)}$$

Safety with Improved Performance

$$\chi_i(z) \triangleq \min\left(\bar{d}\left\|\nabla B_i(x)\right\|, \nabla B_i^T(x)\,\hat{d} + \tilde{d}_{UB}(t)\left\|\nabla B_i(x)\right\|\right)$$

$$\tilde{d}_{UB}(t) \triangleq \bar{d}\,e^{-\lambda t}$$



Implementation

$$\kappa^*(z) \triangleq \underset{u \in \mathbb{R}^m}{\arg\min}\; Q(x,u)$$
$$\text{s.t. } \bar{\Gamma}(z,u) \leq -\gamma(x)$$

$$\bar{\Gamma}_i(z,u) \triangleq \nabla B_i^T(x)\left(f(x) + g(x)u\right) + \chi_i(z)$$

- We provide a result certifying local Lipschitz continuity of the control law when
  - Optimization is convex with unique minimizer for fixed z
  - Constraints and cost are twice continuously differentiable
  - The linear independence constraint qualification holds at z

Modular Adaptive Safety

- Can now integrate feedforward estimates of dynamics with guaranteed safety

$$\dot{x} = \hat{f}_k(x) + g(x)u + \tilde{f}_k(x)$$

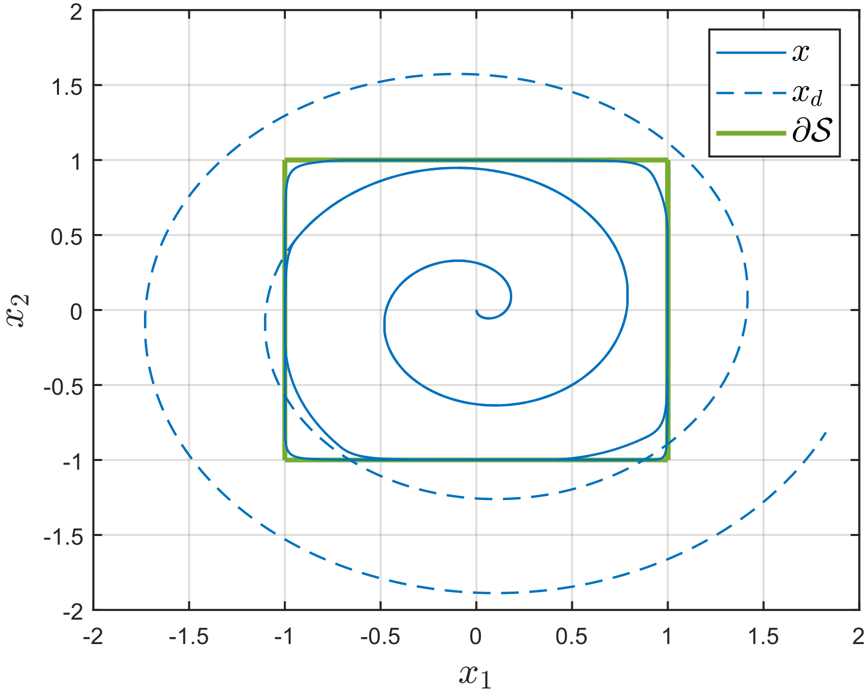
$d(x)$

- Beneficial because RISE gain conditions are reduced through smaller uncertainty

Simulation

- Pre-trained DNN used to approximate dynamics

- RISE-based observer eliminates the residual estimation error



$$\dot{x} = \begin{bmatrix} \cos(x_1)\sin(x_2)\tanh(x_2) + \operatorname{sech}^2(x_1) \\ \operatorname{sech}^2(x_1) \end{bmatrix} + u$$

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Future Directions

- The developed framework can accommodate any estimator that provides upper bounds of the estimation error

- Disturbance estimation error analysis that accounts for measurement error and discrete-time implementation
    - Need upper bounds that are robust to these factors

Previous Work

$$\dot{x} = Y(x,t)\,\theta + g(x)\,u$$

$$\tilde{\theta} \triangleq \theta - \hat{\theta}$$

Integral Concurrent Learning

Using the update law,

$$\dot{\hat{\theta}} \triangleq k_{CL} \sum_{i=1}^{N(t)} \mathcal{Y}_i^T \left(\phi(t_i) - \phi(t_i - \Delta t) - \mathcal{K}_i - \mathcal{Y}_i \hat{\theta} \right)$$

$$\mathcal{Y}(t) \triangleq \int_{t-\Delta t}^{t} Y(\phi(\tau), \tau)\, \mathrm{d}\tau$$

Leads to,

$$\dot{\tilde{\theta}} = -k_{CL} \sum_{i=1}^{N(t)} \mathcal{Y}_i^T \mathcal{Y}_i \tilde{\theta}$$

Compensation for Estimation Error

We show that,

$$\left\|\tilde{\theta}(t)\right\| \leq \tilde{\theta}_{UB}(t)$$

for all $t \in \operatorname{dom} \phi$, where

$$\tilde{\theta}_{UB}(t) \triangleq \left\|\tilde{\theta}(0)\right\| \exp\left(-\int_0^t k_{CL} \lambda_{min}(\tau)\, \mathrm{d}\tau \right)$$

$$\lambda_{min}\left\{ \sum_{i=1}^{N(t)} \mathcal{Y}_i^T \mathcal{Y}_i \right\}$$

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Previous Work

$$\dot{x} = Y(x,t)\,\theta + g(x)\,u$$

$$\tilde{\theta} \triangleq \theta - \hat{\theta}$$

Observation 1

When θ is bounded, $\tilde{\theta}_{UB}(t)$ is computable

Observation 2

When $\lambda_{min} \geq \underline{\lambda} > 0$ for all $t \geq T$, then $\tilde{\theta}_{UB}(t)$ is exponentially regulated

Compensation for Estimation Error

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for all $t \in \operatorname{dom} \phi$, where

$$\tilde{\theta}_{UB}(t) \triangleq \left\|\tilde{\theta}(0)\right\| \exp\left(-\int_0^t k_{CL}\lambda_{min}(\tau)\,\mathrm{d}\tau\right)$$

$$\lambda_{min}\left\{\sum_{i=1}^{N(t)} \mathcal{Y}_i^T \mathcal{Y}_i\right\}$$

Previous Work

$$\nabla B_i^T(x)\, Y(x,t)\,\theta = \nabla B_i^T(x)\left(Y(x,t)\,\hat{\theta} + Y(x,t)\,\tilde{\theta}\right)$$

$$\left\|\tilde{\theta}(t)\right\| \leq \tilde{\theta}_{UB}(t)$$

Compensate for estimation error using:

$$\nabla B_i^T(x)\, Y(x,t)\,\tilde{\theta}(t) \leq \left\|\nabla B_i^T(x)\, Y(x,t)\right\| \tilde{\theta}_{UB}(t)$$

$$\chi_i(z) \triangleq \min\left(\left\|\nabla B_i^T(x)\, Y(x,t)\right\| \bar{\theta}, \nabla B_i^T(x)\, Y(x,t)\,\hat{\theta}(t) + \left\|\nabla B_i^T(x)\, Y(x,t)\right\| \tilde{\theta}_{UB}(t)\right)$$

$$\bar{\Gamma}_i(z,u) \triangleq \nabla B_i^T(x)\left(f(x) + g(x)\,u\right) + \chi_i(z)$$

Thanks!