

Distributed RISE-Based Control for Exponential Heterogeneous Multi-Agent Target Tracking of Second-Order Nonlinear Systems

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Abstract—A distributed Robust Integral of the Sign of the Error (RISE) controller is developed for multi-agent target tracking with exponential convergence. Previous RISE-based multi-agent approaches required 2-hop communication, limiting practical applicability. New insights from a Lyapunov-based design-analysis approach eliminate this multi-hop requirement, yielding exponential target tracking. These insights include a new P -function that works in tandem with the graph interaction matrix in the Lyapunov function. Nonsmooth Lyapunov-based stability analysis yields semi-global exponential convergence to the target agent state despite bounded disturbances. The resulting controller achieves exponential target tracking with only local information exchange between neighboring agents.

Index Terms—Networked control systems, robust control, adaptive control.

I. INTRODUCTION

MULTI-AGENT systems have emerged as a powerful framework for distributed sensing, decision-making, and control across various domains. A challenging problem within this framework is target tracking, where networked agents must collectively track a designated target despite uncertainties and limited information exchange [1], [2]. This capability enables critical applications in surveillance, search and rescue, environmental monitoring, and defense systems [3], [4], [5].

The multi-agent target tracking problem presents several technical challenges, including network topology constraints, communication limitations, and the presence of unmodeled dynamics and external disturbances. Robust control techniques that effectively handle these uncertainties while maintaining stability guarantees are therefore essential. Among these

techniques, the Robust Integral of the Sign of the Error (RISE) methodology has demonstrated significant potential by achieving exponential tracking error convergence with continuous control inputs despite time-varying disturbances [6], [7], [8].

The RISE control approach incorporates integration of a signum term to achieve robust performance, making it particularly suitable for multi-agent systems where disturbance rejection is critical. Central to the RISE control structure is the design of the P -function, which is used in the Lyapunov-based stability analysis. The P -function construction leverages convolutional integral properties and specific 1-norm identities. However, extending the traditional P -function construction used in previous literature to multi-agent systems presents significant challenges, because the stability analysis is typically conducted at the ensemble level rather than the agent level.

The first application of RISE to multi-agent tracking was presented in [9], where a synchronization error is used to represent the collective tracking error, which includes an interaction matrix to combine the graph Laplacian and pinning matrix. The controller in [9] requires 2-hop communication, meaning agents need information from the neighbors of their neighboring agents, which may limit some practical scenarios. RISE control has been applied to related multi-agent problems, including the leader-following flocking problem [10], where a virtual leader guides the multi-agent system, thereby avoiding challenges inherent to target tracking where targets may be indifferent or noncooperative. Additionally, [11] employed RISE control for distributed stabilization in multi-agent systems with uncertain strong-weak competition networks, addressing a fundamentally different problem from target tracking.

A distributed implementation of a RISE-based controller for multi-agent target tracking is developed in this letter. The key innovation is the inclusion of the graph interaction matrix directly in the Lyapunov function, rather than coupling it explicitly to the signum term as in [9]. This modification eliminates the need for 2-hop communication but introduces new challenges in the P -function construction. To overcome these challenges, a novel P -function design is developed that incorporates the interaction matrix while enabling the use of 1-norm identities. Through a nonsmooth stability analysis, semi-global exponential convergence to the target agent state is established.

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The primary contributions of this letter are: (1) a distributed RISE-based control design that requires only local information exchange, (2) a novel P -function construction technique for multi-agent systems with interaction matrix incorporation, and (3) exponential convergence despite the presence of disturbances and uncertainties.

II. PRELIMINARIES

A. Notation

For a set A and an input x , the indicator function is denoted by $\mathbf{1}_A(x)$, where $\mathbf{1}_A(x) = 1$ if $x \in A$, and $\mathbf{1}_A(x) = 0$ otherwise. For a vector $\mathbf{x} \in \mathbb{R}^n$, the element-wise signum function is given by $\text{sgn}(\mathbf{x}) = [\text{sgn}(x_1) \cdots \text{sgn}(x_n)]^\top \in \mathbb{R}^n$, where each $x_i \in \mathbb{R}$ and $\text{sgn}(x_i) = \mathbf{1}_{\{x_i > 0\}}(x_i) - \mathbf{1}_{\{x_i < 0\}}(x_i)$. Let $\mathbf{1}_n \in \mathbb{R}^n$ denote the column vector of length $n > 1$ whose entries are all ones. Similarly, $\mathbf{0}_n \in \mathbb{R}^n$ denotes the column vector of length $n > 1$ whose entries are all zeroes. For $m, n > 1$, $\mathbf{0}_{m \times n} \in \mathbb{R}^{m \times n}$ denotes the $m \times n$ zero matrix. The $p \times p$ identity matrix is denoted by I_p . Given $M \in \mathbb{Z}_{>0}$, define $[M] \triangleq \{1, 2, \dots, M\}$. Let $n, m \in \mathbb{Z}_{>0}$ with $m > n$. The p -norm of $r \in \mathbb{R}^n$ is denoted by $\|r\|_p$, where in the special case that $p = 2$, $\|\cdot\|$ is used. The Euclidean norm of $r \in \mathbb{R}^n$ is $\|r\| \triangleq \sqrt{r^\top r}$. Given a positive integer N and collection $\{x_i\}_{i \in [N]} \subset \mathbb{R}^n$, let $(x_i)_{i \in [n]} \triangleq [x_1^\top \cdots x_N^\top]^\top \in \mathbb{R}^{nN}$. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a differentiable function. The maximum and minimum eigenvalues of $A = A^\top$ are denoted by $\bar{\lambda}_A \in \mathbb{R}$ and $\underline{\lambda}_A \in \mathbb{R}$, respectively.

B. Nonsmooth Analysis

The space of essentially bounded Lebesgue measurable functions is denoted by $\mathcal{L}_\infty$. A function is of class C^k if it has k continuous derivatives. The Lebesgue measure on $\mathbb{R}^n$ is denoted by μ . Given some sets A and B , a set-valued map F from A to subsets of B is denoted by $F : A \rightrightarrows B$. The notation $\text{co}A$ denotes the closed convex hull of the set A . The notation $\mathbb{B}_\delta(x)$, for $x \in \mathbb{R}^n$ and $\delta > 0$, is used to denote the set $\{y \in \mathbb{R}^n : \|x - y\| < \delta\}$. The interior of a set S , denoted by $\text{int}(S)$, is the set of all points $x \in S$ for which there exists $\delta > 0$ such that $\mathbb{B}_\delta(x) \subset S$. The closure of a set S , denoted by $\bar{S}$, is the union of S and its boundary points. A set is compact if it is closed and bounded, and a set is precompact if its closure is compact. Consider a Lebesgue measurable and locally essentially bounded function $h : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$. The Filippov regularization of h is defined as $K[h](y, t) \triangleq \bigcap_{\delta > 0} \bigcap_{\mu \mathbb{S} = 0} \text{co} h(\mathbb{B}_\delta(y) \setminus \mathbb{S}, t)$, where $\bigcap_{\mu \mathbb{S} = 0}$ denotes the intersection over all sets $\mathbb{S}$ of Lebesgue measure zero [12, Equation (2)b]. Additionally, given any sets $A, B \subset \mathbb{R}$, the notation $A \leq B$ is used to state $a \leq b$ for all $a \in A$ and $b \in B$. The notation “a.e.” (almost everywhere) means that a property P holds for all $x \in X \setminus N$, where $N \in \Sigma$ and $\mu(N) = 0$ in a measure space (X, Σ, μ) . A function $y : \mathcal{I}_y \rightarrow \mathbb{R}^n$ is called a Filippov solution of $\dot{y} = h(y, t)$ on the interval $\mathcal{I}_y \subseteq \mathbb{R}_{\geq 0}$, if y is absolutely continuous on $\mathcal{I}_y$, and is a solution to the differential inclusion $\dot{y} \stackrel{\text{a.e.}}{\in} K[h](y, t)$. Clarke’s generalized gradient for a locally Lipschitz function $V : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ is defined as $\partial V(x, t) \triangleq \text{co}\{\lim \nabla V(x, t) : (x_i, t_i) \rightarrow (x, t), (x_i, t_i) \notin \Omega_V\}$, where Ω_V denotes the set of measure zero wherever ∇V is not defined [13, Def. 2.2].

C. Algebraic Graph Theory

Consider a network of N distinct vertices indexed by $i \in \mathcal{V}$, where $\mathcal{V} \triangleq [N]$ for $N \in \mathbb{Z}_{>0}$. The flow of information between vertices is modeled by a static, weighted, and undirected graph $G \triangleq (\mathcal{V}, E, w)$, where $E \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges between distinct vertices, and $w : E \rightarrow \mathbb{R}_{>0}$ is a function that assigns a positive real weight $w(e)$ to each edge $e \in E$. A bidirectional exchange of information between vertices i and j , denoted by $i \leftrightarrow j$, exists if and only if $(i, j) \in E$ and $(j, i) \in E$ with $i \neq j$. The neighborhood of vertex i , denoted by $\mathcal{N}_i$, is the set of vertices $j \in \mathcal{V}$ such that information flows between i and j , i.e., $\mathcal{N}_i \triangleq \{j \in \mathcal{V} : (j, i) \in E \wedge (i, j) \in E, j \neq i\}$. The adjacency matrix $A \in \mathbb{R}^{N \times N}$ of the graph G is defined by $a_{ij} \triangleq w((j, i)) \mathbf{1}_{\{(j, i) \in E \wedge (i, j) \in E\}}$, the degree matrix $D \in \mathbb{R}^{N \times N}$ is defined by $D \triangleq \text{diag}\{A \mathbf{1}_N\}$, and the graph Laplacian $L \in \mathbb{R}^{N \times N}$ is defined by $L \triangleq D - A$.

Consider an external vertex indexed by 0, and define the extended vertex set as $\bar{\mathcal{V}} \triangleq \mathcal{V} \cup \{0\}$. The extended flow of information is modeled by a static graph $\bar{G} \triangleq (\bar{\mathcal{V}}, \bar{E})$, where $\bar{E} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ is the extended edge set. Exclusive to vertex 0, the notation $0 \rightarrow i$ indicates that vertex i can sense vertex 0, i.e., $0 \rightarrow i$ exists if and only if $(0, i) \in \bar{E}$. If $0 \rightarrow i$, vertex i is said to be pinned.

III. PROBLEM FORMULATION

A. System Dynamics

Consider a multi-agent system composed of N agents indexed by $\mathcal{V}$, and consider a target agent indexed by $\{0\}$. The dynamics for agent $i \in \mathcal{V}$ are given by¹

$$\ddot{q}_i = f_i(q_i, \dot{q}_i, t) + g_i(q_i, \dot{q}_i, t)u_i(t) + d_i(t), \quad (1)$$

where: $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ denote the agents’ unknown generalized position, velocity, and acceleration, respectively; the unknown functions $f_i : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n$ and $d_i : [0, \infty) \rightarrow \mathbb{R}^n$ represent modeling uncertainties and exogenous disturbances, respectively; $g_i : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^{n \times m_i}$ denotes a known control effectiveness matrix; and $u_i : [0, \infty) \rightarrow \mathbb{R}^{m_i}$ denotes the control input. Here, the functions f_i are of class C^3 , the mappings $t \mapsto f_i(q_i, \dot{q}_i, t)$, $t \mapsto \nabla f_i(q_i, \dot{q}_i, t)$, and $t \mapsto \nabla^2 f_i(q_i, \dot{q}_i, t)$ are uniformly bounded, the functions g_i have full row-rank, are of class C^3 , the mappings $t \mapsto g_i(q_i, \dot{q}_i, t)$, $t \mapsto \nabla g_i(q_i, \dot{q}_i, t)$, and $t \mapsto \nabla^2 g_i(q_i, \dot{q}_i, t)$ are uniformly bounded, and the functions d_i are of class C^2 , and there exist known constants $\bar{d}_i, \bar{\dot{d}}_i, \bar{\ddot{d}}_i \in \mathbb{R}_{>0}$ such that $\|d_i(t)\| \leq \bar{d}_i$, $\|\dot{d}_i(t)\| \leq \bar{\dot{d}}_i$, and $\|\ddot{d}_i(t)\| \leq \bar{\ddot{d}}_i$ for all $t \in [0, \infty)$, for all $i \in \mathcal{V}$. By the full row-rank property, the existence of the right Moore-Penrose inverse $g_i^+ : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^{m_i \times n}$ is ensured, for all $i \in \mathcal{V}$, where the functions g_i^+ are of class C^1 , and the mappings $t \mapsto g_i^+(q_i, \dot{q}_i, t)$ and $t \mapsto \nabla g_i^+(q_i, \dot{q}_i, t)$ are uniformly bounded for all $i \in \mathcal{V}$.

The dynamics for the target agent are given by

$$\ddot{q}_0 = f_0(q_0, \dot{q}_0, t), \quad (2)$$

where $q_0, \dot{q}_0, \ddot{q}_0 \in \mathbb{R}^n$ denote the target’s unknown generalized position, velocity, and acceleration, respectively, and the function $f_0 : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n$ is unknown, of class C^3 ,

¹For addressing an uncertain g_i , potential extensions might leverage techniques found in [14], [15].

and the mappings $t \mapsto \nabla f_0(q_0, \dot{q}_0, t)$ and $t \mapsto \nabla^2 f_0(q_0, \dot{q}_0, t)$ are uniformly bounded.

Assumption 1: There exist known constants $\bar{q}_0, \bar{\dot{q}}_0, \bar{\ddot{q}}_0, \bar{q}_0 \in \mathbb{R}_{>0}$ such that $\|q_0(t)\| \leq \bar{q}_0$, $\|\dot{q}_0(t)\| \leq \bar{\dot{q}}_0$, $\|\ddot{q}_0(t)\| \leq \bar{\ddot{q}}_0$, and $\|\ddot{q}_0(t)\| \leq \bar{q}_0$ for all $t \in [0, \infty)$.

B. Control Objective

Each agent $i \in \mathcal{V}$ can measure the relative position $q_{i,j} \in \mathbb{R}^n$ and relative velocity $\dot{q}_{i,j} \in \mathbb{R}^n$ between itself and any agent $j \in \bar{\mathcal{N}}_i$, where the relative position is defined as

$$q_{ij} \triangleq q_j - q_i. \quad (3)$$

The objective is to design a distributed controller that enables all agents to track the target using only relative state measurements, given the unknown dynamics described by (1) and (2). To quantify the tracking performance, the target tracking error $e_i \in \mathbb{R}^n$ is defined as

$$e_i \triangleq q_0 - q_i. \quad (4)$$

Using (4), the neighborhood tracking error $\eta_i \in \mathbb{R}^n$ is defined as

$$\eta_i \triangleq b_i e_i + \sum_{j \in \bar{\mathcal{N}}_i} a_{ij} q_{ij}, \quad (5)$$

where $b_i \in \{0, 1\}$ indicates whether agent $i \in \mathcal{V}$ senses the target. Using (3), (5) is expressed in an equivalent analytical form as

$$\eta_i = b_i e_i - \sum_{j \in \bar{\mathcal{N}}_i} a_{ij} (e_j - e_i). \quad (6)$$

IV. CONTROL DESIGN

Let $\mathcal{I} \triangleq [t_0, t_1]$ denote the interval where solutions exist for the subsequent closed-loop error system, where $t_0, t_1 \in \mathbb{R}_{\geq 0}$ and $t_1 > t_0$. Based on the subsequent stability analysis, the continuous control input u_i for each agent $i \in \mathcal{V}$ is designed as

$$u_i = g_i^+(q_i, \dot{q}_i, t)(k_3 \dot{\eta}_i + ((k_1 + k_2)k_3 + 1)\eta_i + (k_1 + k_2)b_i \dot{e}_i + (1 + k_1 k_2)b_i e_i + \hat{v}_i) \quad (7)$$

where $\hat{v}_i \in \mathbb{R}^n$ is designed as a Filippov solution to

$$\dot{\hat{v}}_i = (k_1 + (1 + k_1 k_2)k_3)\eta_i + k_4 \text{sgn}(\dot{\eta}_i + k_1 \eta_i), \quad (8)$$

where $k_1, k_2, k_3, k_4 \in \mathbb{R}_{>0}$ are user-defined constants. To aid in the stability analysis, the graph interaction matrix $\mathcal{H} \in \mathbb{R}^{nN \times nN}$ is defined as

$$\mathcal{H} \triangleq (L + B) \otimes I_n, \quad (9)$$

where $B \triangleq \text{diag}\{b_1, \dots, b_N\} \in \mathbb{R}^{N \times N}$. Using (9), the neighborhood position error in (6) is expressed in an ensemble representation as

$$\eta = \mathcal{H}e, \quad (10)$$

where $\eta \triangleq (\eta_i)_{i \in \mathcal{V}} \in \mathbb{R}^{nN}$ and $e \triangleq (e_i)_{i \in \mathcal{V}} \in \mathbb{R}^{nN}$. Based on the subsequent stability analysis, define the filtered tracking error $r_1 \in \mathbb{R}^{nN}$ as

$$r_1 \triangleq \dot{e} + k_1 e. \quad (11)$$

Using (10), (11) is expressed as

$$\mathcal{H}r_1 = \dot{\eta} + k_1 \eta. \quad (12)$$

Similarly, define the auxiliary tracking error $r_2 \in \mathbb{R}^{nN}$ as

$$r_2 \triangleq \dot{r}_1 + k_2 r_1 + e. \quad (13)$$

Using (12), (13) is expressed as

$$\mathcal{H}r_2 = \ddot{\eta} + (k_1 + k_2)\dot{\eta} + (1 + k_1 k_2)\eta. \quad (14)$$

Using (8), (12), and (14), the time-derivative of (7) is expressed in an ensemble representation as

$$\dot{u} = g^+(\mathcal{H}(r_1 + k_3 r_2) + k_4 \text{sgn}(\mathcal{H}r_1) + \mathcal{B}(k_1 \ddot{e} + k_2 \dot{r}_1 + \dot{e})), \quad (15)$$

where $u \triangleq (u_i)_{i \in \mathcal{V}} \in \mathbb{R}^{N \sum_{i \in \mathcal{V}} m_i}$, $g^+ \triangleq \text{diag}(g_1^+, \dots, g_N^+) \in \mathbb{R}^{N(\sum_{i \in \mathcal{V}} m_i \times n)}$, and $\mathcal{B} \triangleq B \otimes I_n \in \mathbb{R}^{nN}$. Substituting (15) into the time-derivative of (13) yields

$$\begin{aligned} \dot{r}_2 = & h_B + \tilde{h} - k_4 \text{sgn}(\mathcal{H}r_1) - (k_3 \mathcal{H} + (k_1 + k_2)(\mathcal{B} - I_{nN}))r_2 \\ & - \left(\mathcal{H} + (1 - 2k_1^2 - k_1 k_2)(\mathcal{B} - I_{nN}) \right) r_1 \\ & + \left(k_1(2 - k_1^2) + k_2 \right) (\mathcal{B} - I_{nN})e, \end{aligned} \quad (16)$$

where $h_B \triangleq (\dot{f}_0(q_0, \dot{q}_0, t) - \dot{f}_i(q_0, \dot{q}_0, t) - \dot{d}_i(t) - \dot{g}_i(q_0, \dot{q}_0, t)g_i^+(q_0, \dot{q}_0, t)(\ddot{q}_0 - f_i(q_0, \dot{q}_0, t) - d_i(t)))_{i \in \mathcal{V}} \in \mathbb{R}^{nN}$ and $\tilde{h} \triangleq (\dot{f}_i(q_0, \dot{q}_0, t) - \dot{f}_i(q_i, \dot{q}_i, t) + \dot{g}_i(q_0, \dot{q}_0, t)g_i^+(q_0, \dot{q}_0, t)(\ddot{q}_0 - f_i(q_0, \dot{q}_0, t) - d_i(t)) - \dot{g}_i(q_i, \dot{q}_i, t)g_i^+(q_i, \dot{q}_i, t)(\ddot{q}_i - f_i(q_i, \dot{q}_i, t) - d_i(t)))_{i \in \mathcal{V}} \in \mathbb{R}^{nN}$.

Assumption 2 [9, Assumption 3]: The graph G is connected with at least one $b_i > 0$, for all $i \in \mathcal{V}$.

Lemma 1 [2, Remark 1.2]: Following Assumption 2, $\mathcal{H}$ is positive definite.

Define the concatenated state vector $z : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{3nN}$ as $z \triangleq [e^\top r_1^\top r_2^\top]^\top$. Using (11), (13), and (16), the closed-loop error system is expressed as

$$\dot{z} = \begin{bmatrix} r_1 - k_1 e \\ r_2 - k_2 r_1 - e \\ \begin{pmatrix} h_B + \tilde{h} - k_4 \text{sgn}(\mathcal{H}r_1) \\ -(k_3 \mathcal{H} + (k_1 + k_2)(\mathcal{B} - I_{nN}))r_2 \\ -(\mathcal{H} + (1 - 2k_1^2 - k_1 k_2)(\mathcal{B} - I_{nN}))r_1 \\ +(k_1(2 - k_1^2) + k_2)(\mathcal{B} - I_{nN})e, \end{pmatrix} \end{bmatrix}. \quad (17)$$

The following section provides a Lyapunov-based stability analysis to provide exponential tracking error convergence guarantees with the developed controller. Some supporting lemmas are first presented to facilitate the subsequent analysis.

Lemma 2: There exist known constants $\chi_1, \chi_2 \in \mathbb{R}_{>0}$ such that $\|h_B\| \leq \chi_1$ and $\|\tilde{h}\| \leq \chi_2$.²

Proof: By Assumption 1, q_0 is defined over a compact set $\mathcal{Q} \subset \mathbb{R}^n$, where $\mathcal{Q} \triangleq \{u \in \mathbb{R}^n : \|u\| \leq \bar{q}_0\}$. Using the definition of h_B and the triangle inequality yields $\|h_B\| \leq N \|\frac{d}{dt}f_0(q_0, \dot{q}_0, t) - \frac{d}{dt}f_i(q_0, \dot{q}_0, t) - \frac{d}{dt}d_i(t) - \frac{d}{dt}(g_i(q_0, \dot{q}_0, t)g_i^+(q_0, \dot{q}_0, t)(f_i(q_0, \dot{q}_0, t) + d_i(t) - \ddot{q}_0))\|$. Applying Assumption 1, the chain-rule, the Cauchy-Schwarz inequality, and the triangle inequality

²If uncertainty bounds χ_1, χ_2 are unknown, adaptive RISE methods (e.g., [16]) can estimate related bounds online. This typically involves a trade-off, as such adaptive approaches often yield asymptotic stability or ultimate boundedness, contrasting with the exponential stability targeted herein, which generally remains an open challenge for adaptive RISE.

yields $\|h_B\| \leq N(\bar{q}_0(\|\frac{\partial f_0}{\partial q_0}\| + \|\frac{\partial f_i}{\partial q_0}\|) + \bar{q}_0(\|\frac{\partial f_0}{\partial q_0}\| + \|\frac{\partial f_i}{\partial q_0}\|) + \|\frac{\partial f_0}{\partial t}\| + \|\frac{\partial f_i}{\partial t}\| + \bar{d}_i + (\|\frac{\partial g_i}{\partial q_0}\|\bar{q}_0 + \|\frac{\partial g_i}{\partial q_0}\|\bar{q}_0 + \|\frac{\partial g_i}{\partial t}\|)g_i^+(q_0, \dot{q}_0, t)(\|f_i(q_0, \dot{q}_0, t)\| + \bar{d}_i + \bar{q}_0))$. Since q_0 is defined over the compact set $\mathcal{Q}$, applying the mean value theorem gives $\|h_B\| \leq \chi_1$.

Using the definition of h_B and the triangle inequality yields $\|\dot{h}_B\| \leq N\|\frac{d^2}{dt^2}f_0(q_0, \dot{q}_0, t) - \frac{d^2}{dt^2}f_i(q_0, \dot{q}_0, t) - \frac{d^2}{dt^2}d_i(t) - \frac{d^2}{dt^2}(g_i(q_0, \dot{q}_0, t))g_i^+(q_0, \dot{q}_0, t)(f_i(q_0, \dot{q}_0, t) + d_i(t) - \bar{q}_0)\|$. Applying Assumption 1, the chain-rule, the product rule, the Cauchy-Schwarz inequality, and the triangle inequality yields $\|\dot{h}_B\| \leq N(\bar{q}_0(\|\frac{d}{dt}(\frac{\partial f_0}{\partial q_0})\| + \|\frac{d}{dt}(\frac{\partial f_i}{\partial q_0})\|) + \bar{q}_0(\|\frac{\partial f_0}{\partial q_0}\| + \|\frac{\partial f_i}{\partial q_0}\|) + \|\frac{\partial f_0}{\partial q_0}\| + \|\frac{d}{dt}(\frac{\partial f_0}{\partial q_0})\| + \bar{q}_0(\|\frac{\partial f_0}{\partial q_0}\| + \|\frac{\partial f_i}{\partial q_0}\|) + \|\frac{d}{dt}(\frac{\partial f_0}{\partial q_0})\| + \|\frac{d}{dt}(\frac{\partial f_i}{\partial q_0})\| + \bar{d}_i + (\|\frac{d}{dt}(\frac{\partial g_i}{\partial q_0})\|\bar{q}_0 + \bar{q}_0(\|\frac{\partial g_i}{\partial q_0}\| + \|\frac{d}{dt}(\frac{\partial g_i}{\partial q_0})\|) + \|\frac{\partial g_i}{\partial q_0}\|\bar{q}_0 + \|\frac{d}{dt}(\frac{\partial g_i}{\partial q_0})\|)g_i^+(q_0, \dot{q}_0, t)(\|f_i(q_0, \dot{q}_0, t)\| + \bar{d}_i + \bar{q}_0))$. Since q_0 is defined over the compact set $\mathcal{Q}$, applying the mean value theorem gives $\|\dot{h}_B\| \leq \chi_2$. ■

Lemma 3 [17, Lemma 5]: There exists a strictly increasing function $\rho: [0, \infty) \rightarrow [0, \infty)$ such that $\|\dot{h}\| \leq \rho(\|z\|)\|z\|$.

V. STABILITY ANALYSIS

To establish exponential stability for the closed-loop error system in (17) a P -function is introduced. This function is used to develop a strict Lyapunov function and is designed to be non-negative under specific gain conditions. The P -function $P: \mathcal{I} \rightarrow \mathbb{R}_{\geq 0}$ is designed as

$$P \triangleq k_4\|\mathcal{H}r_1\|_1 - r_1^\top \mathcal{H}^\top h_B + e^{-\lambda_P t} * r_1^\top \mathcal{H}^\top \dot{h}_B + e^{-\lambda_P t} * (k_4(k_2 - \lambda_P)\|\mathcal{H}r_1\|_1 - (k_2 - \lambda_P)r_1^\top \mathcal{H}^\top h_B), \quad (18)$$

where $\lambda_P \in \mathbb{R}_{>0}$ is a user-defined constant, and the symbol ‘*’ denotes the convolutional integral, defined for any given function $\alpha(t): \mathcal{I} \rightarrow \mathbb{R}$ as $e^{-\lambda_P t} * \alpha(t) = \int_{t_0}^t e^{-\lambda_P(t-\tau)} \alpha(\tau) d\tau$. Using Leibniz’s rule, the time derivative of a convolution integral satisfies $\frac{d}{dt}(e^{-\lambda_P t} * \alpha) = \alpha(t) - \lambda_P \int_{t_0}^t e^{-\lambda_P(t-\tau)} \alpha(\tau) d\tau$ which simplifies to $\frac{d}{dt}(e^{-\lambda_P t} * \alpha) = \alpha(t) - \lambda_P e^{-\lambda_P t} * \alpha(t)$. Using (18) and the definition of the convolutional integral yields

$$P(t_0) = k_4\|\mathcal{H}r_1(t_0)\|_1 - r_1^\top(t_0)\mathcal{H}^\top h_B(t_0). \quad (19)$$

Since $t \mapsto \mathcal{H}r_1(t)$ is absolutely continuous and $\|\cdot\|_1$ is globally Lipschitz, the mapping $t \mapsto \|\mathcal{H}r_1(t)\|_1$ is differentiable almost everywhere. By the chain rule from [13, Th. 2.2], the derivative of $\|\mathcal{H}r_1\|_1$ is given almost everywhere by $\frac{d}{dt}\|\mathcal{H}r_1\|_1 = \frac{d}{dt}\|\dot{\eta} + k_1\eta\|_1 \stackrel{\text{a.e.}}{=} (\dot{\eta} + k_1\eta)^\top K[\text{sgn}](\dot{\eta} + k_1\eta)$. By the definition of the L_1 -norm, $\|\dot{\eta} + k_1\eta\|_1 = (\dot{\eta} + k_1\eta)^\top K[\text{sgn}](\dot{\eta} + k_1\eta)$. Using the definitions of the chain rule and the L_1 -norm, taking the time-derivative of (18), using Leibniz’s rule, and substituting (12), (14), and (18) into the resulting expression yields that $t \mapsto P(t)$ satisfies the differential inclusion

$$\dot{P} \stackrel{\text{a.e.}}{\in} -\lambda_P P - r_2^\top \mathcal{H}^\top (h_B - k_4 K[\text{sgn}](\mathcal{H}r_1)). \quad (20)$$

To facilitate the inclusion of the P -function into the subsequent Lyapunov function candidate, P must be designed to be non-negative under certain sufficient gain conditions.

Proposition 1: For the P -function defined as in (18), if $k_4 > \chi_1 + \frac{\chi_2}{k_2 - \lambda_P}$ with $\lambda_P \in (0, k_2)$, then $P(t) \geq 0$, for all $t \in \mathcal{I}$.

Proof: By Lemma 2, the $k_4\|\mathcal{H}r_1\|_1 - r_1^\top \mathcal{H}^\top h_B$ term in (18) satisfies $k_4\|\mathcal{H}r_1\|_1 - r_1^\top \mathcal{H}^\top h_B \geq (k_4 - \chi_1)\|\mathcal{H}r_1\|_1 \geq 0$ by the Cauchy-Schwarz inequality and the condition $k_4 > \chi_1$. The convolution integrand in (18) is bounded as $r_1^\top \mathcal{H}^\top \dot{h}_B + (k_2 - \lambda_P)(k_4\|\mathcal{H}r_1\|_1 - r_1^\top \mathcal{H}^\top h_B) \geq \|\mathcal{H}r_1\|_1((k_2 - \lambda_P)(k_4 - \chi_1) - \chi_2)$. This expression is non-negative when $(k_2 - \lambda_P)(k_4 - \chi_1) \geq \chi_2$ which is equivalent to $k_4 \geq \chi_1 + \frac{\chi_2}{k_2 - \lambda_P}$. Since $k_4 > \chi_1 + \frac{\chi_2}{k_2 - \lambda_P}$ by assumption, the convolution integrand is positive, making the entire convolution positive. Therefore, $P(t) \geq 0$, for all $t \in \mathcal{I}$. ■

To state the main results, the following definitions are introduced. Let $W: \mathbb{R}^{3nN} \rightarrow \mathbb{R}_{\geq 0}$ be defined as

$$W(\sigma) \triangleq \frac{1}{\lambda_Q} \sqrt{\lambda_Q \|\sigma\|^2 + 2(k_4 + \chi_1)\|\mathcal{H}\|_1 \|\sigma\|_1}, \quad (21)$$

for all $\sigma \in \mathbb{R}^{3nN}$, where $Q \triangleq \text{diag}(I_{2nN}, \mathcal{H}) \in \mathbb{R}^{3nN \times 3nN}$ and χ_1 is defined as in Lemma 2. Additionally, let $k_{\min} \in \mathbb{R}$ be a constant gain defined as $k_{\min} \triangleq \min\{k_1 - \frac{1}{2}, k_2 - \frac{1}{2}, 2(k_1 + k_2)(\lambda_{\mathcal{H}}\lambda_{(\mathcal{B}-I_{nN})} + k_3\lambda_{\mathcal{H}}^2) - (\bar{\lambda}_{(I_{nN}-\mathcal{H}^2)} + (1 + 2k_1^2 + k_1 k_2)\bar{\lambda}_{\mathcal{H}}\bar{\lambda}_{(\mathcal{B}-I_{nN})})^2 - (k_1(2 - k_1^2) + k_2)^2 \bar{\lambda}_{\mathcal{H}}^2 \bar{\lambda}_{(\mathcal{B}-I_{nN})}^2\}$, $\lambda_V \in \mathbb{R}_{>0}$ be the desired rate of convergence, and the set of stabilizing initial conditions $\mathcal{S} \subset \mathbb{R}^{3nN}$ be defined as

$$\mathcal{S} \triangleq \left\{ \sigma \in \mathbb{R}^{3nN} : \rho(W(\sigma)) \leq \frac{k_{\min} - \lambda_V}{\bar{\lambda}_{\mathcal{H}}} \right\}. \quad (22)$$

Let $\xi: \mathcal{I} \rightarrow \mathbb{R}^{3nN+1}$ be defined as $\xi(t) = [z^\top(t) P(t)]^\top$, and $\psi: \mathbb{R}^{3nN+1} \times \mathbb{R}_{\geq 0} \Rightarrow \mathbb{R}^{3nN+1}$ denote the set-valued map

$$\psi(\xi, t) \triangleq \begin{bmatrix} r_1 - k_1 e \\ r_2 - k_2 r_1 - e \\ \begin{pmatrix} h_B + \dot{h} - k_4 K[\text{sgn}](\mathcal{H}r_1) \\ -(k_1 + k_2)(\mathcal{B} - I_{nN})r_2 \\ -(1 - 2k_1^2 - k_1 k_2)(\mathcal{B} - I_{nN})r_1 \\ -\mathcal{H}r_1 - k_3 \mathcal{H}r_2 \\ + (k_1(2 - k_1^2) + k_2)(\mathcal{B} - I_{nN})e \end{pmatrix} \\ \begin{pmatrix} -\lambda_P P - r_2^\top \mathcal{H}^\top h_B \\ + k_4 r_2^\top \mathcal{H}^\top K[\text{sgn}](\mathcal{H}r_1) \end{pmatrix} \end{bmatrix}. \quad (23)$$

Then using (17) and (20), it follows that the trajectories $t \mapsto \xi(t)$ satisfy the differential inclusion $\dot{\xi} \stackrel{\text{a.e.}}{\in} \psi(\xi, t)$.

Theorem 1: All solutions to (17) with $z(t_0) \in \text{int}(\mathcal{S})$ satisfy $\|z(t)\| \leq W(z(t_0))e^{-\frac{\lambda_V}{\lambda_Q}(t-t_0)}$, for all $t \in [t_0, \infty)$, provided that the sufficient control gains k_1, k_2, k_3 are selected to satisfy $k_1 > \frac{1}{2}$, $k_2 > \frac{1}{2}$, $k_3 > (\bar{\lambda}_{(I_{nN}-\mathcal{H}^2)} + \bar{\lambda}_{\mathcal{H}}\bar{\lambda}_{(\mathcal{B}-I_{nN})})^2 + 2\bar{\lambda}_{\mathcal{H}}^2 \bar{\lambda}_{(\mathcal{B}-I_{nN})}^2$, k_4 is selected to satisfy Proposition 1, and Assumptions 1 and 2 hold.

Proof: Consider the function $V: \mathbb{R}^{3nN+1} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ defined as

$$V(\xi) \triangleq \frac{1}{2} z^\top Q z + P. \quad (24)$$

For the subsequent analysis, consider all arbitrary trajectories $t \mapsto \xi(t)$ satisfying (19) and $z(t_0) \in \text{int}(\mathcal{S})$. Since Proposition 1 holds, it follows that these trajectories satisfy $P(t) \geq 0$ for all $t \in \mathcal{I}$ which implies that $V(\xi(t)) > 0$ for all $t \in \mathcal{I}$. Invoking the Rayleigh quotient theorem and using (24) yields

$$\frac{\lambda_Q}{2} \|z\|^2 + P \leq V(\xi). \quad (25)$$

Based on the chain rule for differential inclusions in [13, Th. 2.2], the derivative of $t \mapsto V(\xi(t))$ exists almost everywhere and is a solution to $\dot{V}(\xi) \stackrel{\text{a.e.}}{\in} \tilde{V}(\xi)$, where the set $\tilde{V}(\xi)$ is defined as $\tilde{V}(\xi) \triangleq \bigcap_{\zeta \in \partial V(\xi)} \zeta^\top \psi(\xi, t)$. Since

$V(\xi)$ is continuously differentiable for all $\xi \in \mathbb{R}^{3nN+1}$, Clarke's gradient reduces to the singleton $\{\nabla V(\xi)\} = \{[e^\top r_1^\top r_2^\top \mathcal{H} \ 1]^\top\}$. Thus, $\tilde{V}(\xi) = \bigcap_{\zeta \in \partial V(\xi)} \zeta^\top \psi(\xi, t) =$

$(\nabla V(\xi))^\top \psi(\xi, t)$. Evaluating $\tilde{V}(\xi)$ yields

$$\begin{aligned} \tilde{V}(\xi) = & -k_1 e^\top e - k_2 r_1^\top r_1 - \lambda_P P + r_2^\top \mathcal{H} \tilde{h} \\ & - r_2^\top \left((k_1 + k_2) \mathcal{H}(\mathcal{B} - I_{nN}) + k_3 \mathcal{H}^2 \right) r_2 \\ & + r_2^\top \left(I_{nN} - \mathcal{H}^2 - (1 - 2k_1^2 - k_1 k_2) \mathcal{H}(\mathcal{B} - I_{nN}) \right) r_1 \\ & + \left(k_1 (2 - k_1^2) + k_2 \right) r_2^\top \mathcal{H}(\mathcal{B} - I_{nN}) e \\ & + k_4 r_2^\top \mathcal{H}^\top (K[\text{sgn}](\mathcal{H} r_1) - K[\text{sgn}](\mathcal{H} r_1)). \end{aligned} \quad (26)$$

By [9, Footnote 2], the right-hand side of (26) is continuous almost everywhere. Specifically, it is discontinuous only on a set of times with Lebesgue measure zero, where $K[\text{sgn}](\mathcal{H} r_1) - K[\text{sgn}](\mathcal{H} r_1) \neq 0$. Since this set is Lebesgue negligible, $K[\text{sgn}](\mathcal{H} r_1) \stackrel{\text{a.e.}}{=} \{\text{sgn}(\mathcal{H} r_1)\}$. Furthermore, from Lemma 3, $\|\tilde{h}\| \leq \rho(\|z\|)\|z\|$. Applying the Cauchy-Schwarz inequality, the triangle inequality, and Young's inequality yields that (26) is upper bounded as

$$\begin{aligned} \tilde{V}(\xi) \stackrel{\text{a.e.}}{\leq} & -\left(k_1 - \frac{1}{2}\right) \|e\|^2 - \left(k_2 - \frac{1}{2}\right) \|r_1\|^2 \\ & - (k_1 + k_2) \left(\bar{\lambda}_{\mathcal{H}} \bar{\lambda}_{(\mathcal{B} - I_{nN})} + k_3 \bar{\lambda}_{\mathcal{H}}^2 \right) \|r_2\|^2 \\ & + \left(\bar{\lambda}_{(I_{nN} - \mathcal{H}^2)} + (1 + 2k_1^2 + k_1 k_2) \bar{\lambda}_{\mathcal{H}} \bar{\lambda}_{(\mathcal{B} - I_{nN})} \right)^2 \|r_2\|^2 \\ & + \frac{1}{2} \left(k_1 (2 - k_1^2) + k_2 \right)^2 \bar{\lambda}_{\mathcal{H}}^2 \bar{\lambda}_{(\mathcal{B} - I_{nN})}^2 \|r_2\|^2 \\ & + \bar{\lambda}_{\mathcal{H}} \rho(\|z\|) \|z\|^2 - \lambda_P P. \end{aligned} \quad (27)$$

Since k_1, k_2 , and k_3 are selected to satisfy $k_1 > \frac{1}{2}$, $k_2 > \frac{1}{2}$, and $k_3 > (\bar{\lambda}_{(I_{nN} - \mathcal{H}^2)} + \bar{\lambda}_{\mathcal{H}} \bar{\lambda}_{(\mathcal{B} - I_{nN})})^2 + 2\bar{\lambda}_{\mathcal{H}}^2 \bar{\lambda}_{(\mathcal{B} - I_{nN})}$, it follows that $k_{\min} > 0$. Consequently, using the definition of $\|z\|$ yields that (27) is upper-bounded as

$$\tilde{V}(\xi) \stackrel{\text{a.e.}}{\leq} -(k_{\min} - \bar{\lambda}_{\mathcal{H}} \rho(\|z\|)) \|z\|^2 - \lambda_P P. \quad (28)$$

Since $z(t_0) \in \mathcal{S}$, it follows from (22) that $k_{\min} > \lambda_V + \bar{\lambda}_{\mathcal{H}} \rho(W(z(t_0)))$. Therefore,

$$\tilde{V}(\xi) \stackrel{\text{a.e.}}{\leq} -(\lambda_V + \bar{\lambda}_{\mathcal{H}} \rho(W(z(t_0))) - \rho(\|z\|)) \|z\|^2 - \lambda_P P. \quad (29)$$

By the definition of W in (21), it follows that $\|z(t_0)\| \leq W(z(t_0))$. Because the solution $t \mapsto z(t)$ is continuous, z cannot instantaneously escape $\mathcal{S}$ at t_0 . Therefore, there exists a time interval $\mathcal{I}_{\mathcal{S}}$ satisfying $\mathcal{I}_{\mathcal{S}} \subseteq \mathcal{I}$ such that $z(t) \in \mathcal{S}$ for all $t \in \mathcal{I}_{\mathcal{S}}$, implying $\|z(t)\| < W(z(t_0))$. Because ρ is strictly increasing, $\rho(\|z(t)\|) < \rho(W(z(t_0)))$ for all $t \in \mathcal{I}_{\mathcal{S}}$. Consequently, using (25), selecting $\lambda_P > \lambda_V$ and recalling $\dot{V}(\xi) \stackrel{\text{a.e.}}{\in} \tilde{V}(\xi)$ yields that

$$\dot{V}(\xi) \leq -\frac{2\lambda_V}{\lambda_Q} V(\xi), \quad (30)$$

for all $t \in \mathcal{I}_{\mathcal{S}}$. Solving the differential inequality in (30) and using (25) yields that $\|z\|$ is upper-bounded as

$$\|z(t)\| \leq \sqrt{2\lambda_Q^{-1} V(\xi(t_0))} e^{-\frac{2\lambda_V}{\lambda_Q}(t-t_0)}, \quad (31)$$

for all $t \in \mathcal{I}_{\mathcal{S}}$. Using (19) yields that $V(\xi(t_0)) = \frac{1}{2} \bar{\lambda}_Q \|z(t_0)\|^2 + k_4 \|\mathcal{H} r_1(t_0)\|_1 - r_1^\top(t_0) \mathcal{H}^\top h_B(t_0)$. Since h_B is bounded by Lemma 2, using the Cauchy-Schwarz inequality and the fact that $\|r_1(t_0)\| \leq \|r_1(t_0)\|_1 \leq \|z(t_0)\|_1$ yields

$$\sqrt{2\lambda_Q^{-1} V(\xi(t_0))} \leq W(z(t_0)), \quad (32)$$

where W is defined in (21). Finally, applying (32) to (31)

yields $\|z(t)\| \leq W(z(t_0)) e^{-\frac{2\lambda_V}{\lambda_Q}(t-t_0)}$, for all $t \in \mathcal{I}_{\mathcal{S}}$.

It remains to be shown that $\mathcal{I}_{\mathcal{S}}$ can be extended to $[t_0, \infty)$. Let $t \mapsto \xi(t)$ be a maximal solution to the differential inclusion $\dot{\xi} \stackrel{\text{a.e.}}{\in} \psi(\xi, t)$ with initial conditions satisfying (19) and $z(t_0) \in \text{int}(\mathcal{S})$. From the preceding analysis, $z(t) \in \text{int}(\mathcal{S})$ for all $t \in \mathcal{I}_{\mathcal{S}}$. This implies that $\xi(t) \in \mathcal{D} \triangleq \{\xi \in \mathbb{R}^{3nN+1}; \xi = [\sigma^\top \varsigma]^\top, \sigma \in \text{int}(\mathcal{S}), \varsigma \in \mathbb{R}_{\geq 0}\}$. For any compact subinterval

$\mathcal{J} \subseteq \mathcal{I}_{\mathcal{S}}$, $\|z(t)\| \leq W(z(t_0)) e^{-\frac{2\lambda_V}{\lambda_Q}(t-t_0)}$, for all $t \in \mathcal{J}$. Since W is a locally bounded function and $e^{-\frac{2\lambda_V}{\lambda_Q}(t-t_0)} \leq 1$ for all

$t \geq t_0$, the mapping $t \mapsto z(t)$ is uniformly bounded on $\mathcal{J}$. By definition, $(\xi, t) \mapsto \psi(\xi, t)$ is locally bounded when ξ is bounded. Since $z(t)$ is bounded on $\mathcal{J}$ and $P(t)$ is non-negative, the trajectory $t \mapsto (\xi(t), t)$ is precompact. Furthermore, since the map $(\xi, t) \mapsto \psi(\xi, t)$ is locally bounded, it follows from [18, Remark 3.4] that $\bigcup_{t \in \mathcal{J}} \psi(\xi(t), t)$ is bounded for every compact subinterval $\mathcal{J} \subseteq \mathcal{I}_{\mathcal{S}}$. Therefore, the conditions of [18, Lemma 3.3] are satisfied, guaranteeing that $t \mapsto \xi(t)$ is complete, i.e., $\mathcal{I} = [t_0, \infty)$. Thus, all trajectories satisfying $z(t_0) \in \text{int}(\mathcal{S})$ also satisfy $\|z(t)\| \leq W(z(t_0)) e^{-\frac{2\lambda_V}{\lambda_Q}(t-t_0)}$, for all $t \in [t_0, \infty)$.

Recall that $k_{\min} > \lambda_V + \bar{\lambda}_{\mathcal{H}} \rho(W(z(t_0)))$, implying that the exponential stability result is semi-global [19, Remark 2], as the set of stabilizing initial conditions in (22) can be made arbitrarily large by appropriately adjusting $k_{\min}$ to encompass any $z(t_0) \in \mathbb{R}^{3nN}$.

Because k_4, χ_1 , and λ_V are independent of the initial time t_0 or initial condition $z(t_0)$, the exponential convergence is uniform [20]. Additionally, the convergence and boundedness of $\|z\|$ implies the convergence and boundedness of $\|e\|$, $\|r_1\|$, and $\|r_2\|$. Therefore, since $q_0, \dot{q}_0, \ddot{q}_0 \in \mathcal{L}_\infty$ by Assumption 1, using (4), (6) and (10)-(14) yields that $q_i, \dot{q}_i, \ddot{q}_i \in \mathcal{L}_\infty$, for all $i \in \mathcal{V}$. Thus, $f_i(q_i, \dot{q}_i, t)$ and $g_i(q_i, \dot{q}_i, t)$ are bounded for all $i \in \mathcal{V}$. Therefore, using (1) yields that $u \in \mathcal{L}_\infty$. ■

VI. SIMULATION

Numerical simulations were performed to evaluate the proposed distributed controller. The scenario involved $N = 8$ agents tracking a dynamic target agent, with inter-agent communication following a cycle graph topology. The pinning matrix was $B = \text{diag}\{1, 0, 1, 0, 1, 0, 1, 0\}$. Agent initial positions were sampled from $U(-10, 10)$ m per axis, with zero initial velocities. The target's initial position was sampled from $U(-10, 10)$ m with an initial velocity of $[1 \ -1 \ 0.5]^\top$ m/s. Each agent i adhered to the dynamics in (1) with $f_i = [c_{i,1}(y_i - z_i) + c_{i,2} \tanh(\dot{x}_i t) c_{i,3}(z_i -$

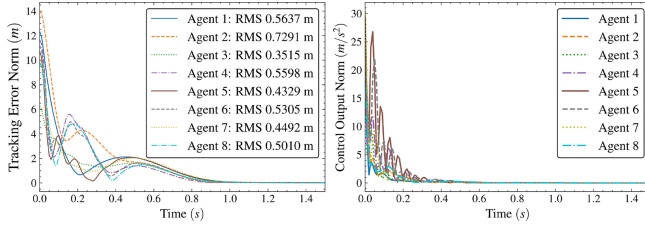


Fig. 1. Left: Tracking error norm for $t \in [0, 1.5]$. Right: Control effort norm for $t \in [0, 1.5]$ s.

$x_i) + c_{i,4} \tanh(\dot{y}_i t) c_{i,5} (x_i - y_i) + c_{i,6} \tanh(\dot{z}_i t)]^\top$, $g_i = I_3 - \text{diag}\{c_{i,7} \cos(t), c_{i,8} \sin(t), c_{i,9} \cos(t) \sin(t)\}$, and $d_i = [c_{i,10} \cos(t) \ c_{i,11} \sin(t) \ c_{i,12} \cos(t) \sin(t)]^\top$ with parameters $c_{i,j}$ sampled from $U(-0.5, 0.5)$. The target's dynamics were $f_0 = [\sin(x_0) - \cos(y_0 \dot{x}_0) \cos(z_0 \dot{y}_0) - \sin(x_0) - \sin(y_0 \dot{z}_0) - \sin(z_0)]^\top$.

Relative position and velocity measurements were corrupted by additive zero-mean Gaussian noise ($\sigma = 0.001$ m and $\sigma = 0.001$ m/s, respectively). Furthermore, heterogeneous time delays, unique to each agent, were introduced: communication delays for inter-agent information, actuation delays for control signal application, and target sensing delays for pinned agents were each randomly sampled for every agent from $U(0.001, 0.02)$ s. Control gains are $k_1 = k_2 = 10$, $k_3 = 25$, and $k_4 = 50$. The total simulation duration was 30 s. Fig. 1 depicts the tracking error norms and control effort norms, demonstrating robust target tracking despite the uncertainties, noise, and varied delays, over a subset of the 30-second simulation.

VII. CONCLUSION

A distributed RISE-based control framework is developed for multi-agent target tracking that achieves exponential convergence while requiring only local information exchange. The key innovation lies in the development of a controller and Lyapunov-based stability analysis which results from incorporating the graph interaction matrix directly into the Lyapunov function, eliminating the need for 2-hop communication present in previous approaches. The control development necessitated the development of a novel P -function construction technique that accommodates the interaction matrix while preserving the mathematical properties required for stability guarantees. The nonsmooth stability analysis established semi-global exponential convergence to the target agent state despite the presence of bounded disturbances with bounded derivatives.

REFERENCES

- [1] F. L. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, *Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches*. London, U.K.: Springer, 2013.
- [2] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks*. London, U.K.: Springer, 2013.
- [3] C. Kwon and I. Hwang, "Sensing-based distributed state estimation for cooperative multiagent systems," *IEEE Trans. Autom. Control*, vol. 64, no. 6, pp. 2368–2382, Jun. 2019.
- [4] Y. Mao, H. Jafarnejadsani, P. Zhao, E. Akyol, and N. Hovakimyan, "Novel stealthy attack and defense strategies for networked control systems," *IEEE Trans. Autom. Control*, vol. 65, no. 9, pp. 3847–3862, Sep. 2020.
- [5] D. S. Drew, "Multi-agent systems for search and rescue applications," *Curr. Robot. Rep.*, vol. 2, no. 2, pp. 189–200, 2021.
- [6] O. Patil, A. Isaly, B. Xian, and W. E. Dixon, "Exponential stability with RISE controllers," *IEEE Control Syst. Lett.*, vol. 6, pp. 1592–1597, 2022.
- [7] O. S. Patil, K. Stubbs, P. Amy, and W. E. Dixon, "Exponential stability with RISE controllers for uncertain nonlinear systems with unknown time-varying state delays," in *Proc. IEEE Conf. Decis. Control*, 2022, pp. 6431–6435.
- [8] J. R. Klotz, R. Kamalapurkar, and W. E. Dixon, "Saturated RISE controllers with exponential stability guarantees: A projected dynamical systems approach," *IEEE Trans. Autom. Control*, early access, Feb. 18, 2025, doi: 10.1109/TAC.2025.3543246.
- [9] J. R. Klotz, Z. Kan, J. M. Shea, E. L. Pasilliao, and W. E. Dixon, "Asymptotic Synchronization of a leader-follower network of uncertain euler-lagrange systems," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 2, pp. 174–182, Jun. 2015.
- [10] X. Wang, J. Sun, Z. Wu, and Z. Li, "Robust integral of sign of error-based distributed flocking control of double-integrator multi-agent systems with a varying virtual leader," *Int. J. Robust Nonlin. Control*, vol. 32, no. 1, pp. 286–303, 2022. [Online]. Available: <https://onlinelibrary.wiley.com/doi/abs/10.1002/rnc.5823>
- [11] H.-X. Hu, G. Wen, X. Yu, Z.-G. Wu, and T. Huang, "Distributed Stabilization of heterogeneous MASs in uncertain strong-weak competition networks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 3, pp. 1755–1767, Mar. 2022.
- [12] B. E. Paden and S. S. Sastry, "A calculus for computing Filippov's differential inclusion with application to the variable structure control of robot manipulators," *IEEE Trans. Circuits Syst.*, vol. 34, no. 1, pp. 73–82, Jan. 1987.
- [13] D. Shevitz and B. Paden, "Lyapunov stability theory of nonsmooth systems," *IEEE Trans. Autom. Control*, vol. 39, no. 9, pp. 1910–1914, Sep. 1994.
- [14] J. Teo, J. P. How, and E. Lavretsky, "On approximate dynamic inversion and proportional-integral control," in *Proc. Am. Control Conf.*, 2009, pp. 1592–1597.
- [15] W. MacKunis, P. Patre, M. Kaiser, and W. E. Dixon, "Asymptotic tracking for aircraft via robust and adaptive dynamic inversion methods," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 6, pp. 1448–1456, Nov. 2010.
- [16] B. Bidikli, E. Tatlicioglu, and E. Zergeroglu, "A self tuning RISE controller formulation," in *Proc. Am. Control Conf.*, 2014, pp. 5608–5613.
- [17] R. Kamalapurkar, J. A. Rosenfeld, J. Klotz, R. J. Downey, and W. E. Dixon, "Supporting lemmas for RISE-based control methods," 2014, *arXiv:1306.3432*.
- [18] R. Kamalapurkar, W. E. Dixon, and A. Teel, "On reduction of differential inclusions and Lyapunov stability," *ESAIM, Control, Optim. Calc. Variat.*, vol. 26, no. 24, pp. 1–16, 2020.
- [19] K. Y. Pettersen, "Lyapunov sufficient conditions for uniform semiglobal exponential stability," *Automatica*, vol. 78, pp. 97–102, Apr. 2018.
- [20] A. Loría and E. Panteley, "Uniform exponential stability of linear time-varying systems: Revisited," *Syst. Control Lett.*, vol. 47, no. 1, pp. 13–24, 2002.