TRACKING CONTROL FOR ROBOT MANIPULATORS WITH KINEMATIC AND DYNAMIC UNCERTAINTY*

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Abstract

The control objective in many robot manipulator applications is to command the end-effector motion to achieve a desired response. To achieve this objective, mapping is required to relate the joint/link control inputs to the desired Cartesian position and orientation. If there are uncertainties or singularities in the mapping, then degraded performance or unpredictable responses by the manipulator are possible. To address these issues, an adaptive tracking controller for robot manipulators with uncertainty in the kinematic and dynamic models is developed in this paper. The controller is developed based on the unit quaternion representation so that singularities associated with three parameter representations are avoided. Experimental results for a planar application of the Barrett whole arm manipulator (WAM) are provided to illustrate the performance of the developed controller.

Key Words

Adaptive control, uncertain kinematics, uncertain dynamics, unit quaternion control

1. Introduction

The control objective in many robot manipulator applications is to command the end-effector motion to achieve a desired response. The control inputs are applied to the manipulator joints and the desired position and orientation are typically encoded in terms of a Cartesian coordinate frame attached to the robot end-effector, with respect to the base frame (that is, the so-called task-space variables). Hence, a mapping (that is, the solution of the inverse kinematics) is required to convert the desired task-space trajectory into a form that can be utilized by the joint space controller. If there are uncertainties or singularities in the mapping, then degraded performance or unpredictable responses by the manipulator can result. Several parametrizations exist to describe orientation angles in the task-space to joint-space mapping, including three-parameter representations (Euler angles, Rodrigues parameters) and the four-parameter representation given by the unit quaternion. Three-parameter representations always exhibit singular orientations (the orientation Jacobian matrix in the kinematic equation is singular for some orientations), while the unit quaternion represents the end-effector orientation without singularities. The emphasis in this paper is to develop a tracking controller that by utilizing the singularity free unit quaternion compensates for uncertainty throughout the kinematic and dynamic models. Some previous task-space control formulations, based on the unit quaternion can, be found in [2–6]. A quaternion-based, resolved acceleration controller was presented in [3] and quaternion-based, resolved rate and resolved acceleration task-space controllers were proposed in [6]. Output feedback task-space controllers, using quaternion feedback, were presented in [4] for the regulation problem and in [2] for the tracking problem. Model-based and adaptive, asymptotic full-state feedback controllers and an output feedback controller based on a model-based observer were developed in [5] using the quaternion parametrization.

A common assumption in most previous robot controllers (including all of the aforementioned quaternion-based, task-space control formulations) is that the robot kinematics and manipulator Jacobian are assumed to be perfectly known. From a review of literature, few controllers have been developed that target uncertainty in the manipulator forward kinematics and Jacobian. For example, in [7–12], several researchers approximate Jacobian feedback controllers, that exploit a static, best-guess estimate of the manipulator Jacobian, to achieve task-space regulation objectives, despite parametric uncertainty in the manipulator Jacobian. In [13], a task-space adaptive controller, for set point control of robots with uncertainties in the gravity regressor matrix and kinematics, was developed. In [14], an adaptive regulation controller, for robot manipulators with uncertainty in the kinematic and

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Recommended by Prof. Gian Luca Foresti

(paper no. 206-3102)

*A preliminary version of this paper appeared in the proceedings of the IEEE Conference on Decision and Control, Seville, Spain, 2005 [1].

dynamic models, was developed. The result in [14] also accounted for actuator saturation, as the maximum commanded torque could be a priori determined, due to the use of saturated feedback terms in the controller. Recently, in [15], an adaptive regulation controller for rigid-link, electrically driven robot manipulators, with uncertainty in kinematics, manipulator dynamics, and actuator dynamics, was developed.

All of the aforementioned controllers that account for kinematic uncertainty are based on the three-parameter, Euler angle representation. Moreover, all of the previous results only target the set-point regulation problem. The only results that target the more general tracking control problem for manipulators with uncertain kinematics are given in [16–18]. These results, however, are also based on the Euler angle representation and, with the exception of [17], they all require the measurement of the task-space velocity. In [17], a filtered derivative of the task-space position is used to generate an approximation of the task-space velocity signal. Hence, motivated by previous work, in this paper an adaptive tracking controller is developed for robot manipulators with uncertainty in the kinematic and dynamic models. The controller is developed based on the unit quaternion representation, so that singularities associated with three parameter representations are avoided. In addition, the developed controller does not require the measurement of the task-space velocity. The stability of the controller is proven through a Lyapunov based stability analysis. Experimental results for a planar application of the Barrett whole arm manipulator (WAM) are provided to illustrate the performance of the developed controller.

2. Robot Dynamic and Kinematic Models

A six-link, rigid, revolute robot manipulator can be described by the following dynamic model [19]

$$M(\theta)\ddot{\theta} + V_m(\theta,\dot{\theta})\dot{\theta} + G(\theta) + F_d\,\dot{\theta} = \tau \qquad (1)$$

In (1), $\theta(t) \in \mathbb{R}^6$ is the joint position[1], $M(\theta) \in \mathbb{R}^{6\times 6}$ represents the inertia matrix, $V_m(\theta,\dot{\theta}) \in \mathbb{R}^{6\times 6}$ is the centripetal-Coriolis matrix, $G(\theta) \in \mathbb{R}^6$ is the gravity vector, $F_d \in \mathbb{R}^{6\times 6}$ is a constant diagonal matrix, which represents the viscous friction coefficients, and $\tau(t) \in \mathbb{R}^6$ represents the input torque vector. The dynamic model given in (1) has the following properties [19], which are utilized in the subsequent control design and analysis.

Property 1 *The inertia matrix is symmetric and positive-definite and satisfies the following inequalities*

$$m_1 \|x\|^2 \le x^T M(\theta) x \le m_2 \|x\|^2 \qquad \forall x \in \mathbb{R}^6 \qquad (2)$$

where $m_1, m_2 \in \mathbb{R}$ are positive constants and $\|\cdot\|$ denotes the standard Euclidean norm.

[1] It is assumed that the actuated manipulator joint is rigidly connected to the links, so that the link-space and joint-space are equivalent. Hence, the words joint and link can be used interchangeably.

Property 2 *The inertia and centripetal-Coriolis matrices satisfy the following skew-symmetric relationship:*

$$x^T\left(\frac{1}{2}\dot{M}(\theta) - V_m(\theta,\dot{\theta})\right)x = 0 \qquad \forall x \in \mathbb{R}^6 \qquad (3)$$

Property 3 *The centripetal-Coriolis matrix satisfies the following skew-symmetric relationship:*

$$V_m(\theta,x)y = V_m(\theta,y)x \qquad \forall x,y \in \mathbb{R}^6 \qquad (4)$$

Property 4 *The norm of the centripetal-Coriolis matrix and the norm of the friction matrix can be upper bounded as follows:*

$$\|V_m(\theta,x)\|_{i\infty} \le \zeta_c \|x\| \qquad \forall x \in \mathbb{R}^6, \quad \|F_d\| \le \zeta_f \qquad (5)$$

where ζ_c, $\zeta_f \in \mathbb{R}$ are positive constants and $\|\cdot\|_{i\infty}$ denotes the induced-infinity norm of a matrix.

Property 5 *Parametric uncertainty in $M(\theta)$, $V_m(\theta,\dot{\theta})$, $G(\theta)$ and F_d is linearly parametrizable.*

Let $\mathcal{E}$ and $\mathcal{B}$ be orthogonal coordinate frames attached to the manipulator's end-effector and fixed base, respectively. The position and orientation of $\mathcal{E}$ relative to $\mathcal{B}$ can be represented through the following forward kinematic model [4]:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} h_p(\theta) \\ h_q(\theta) \end{bmatrix} \qquad (6)$$

In (6), $h_p(\cdot) : \mathbb{R}^6 \to \mathbb{R}^3$ denotes an uncertain function that maps $\theta(t)$ to the measurable[2] task-space position coordinates of the end-effector, denoted by $p(\cdot) \in \mathbb{R}^3$. $h_q(\cdot) : \mathbb{R}^6 \to \mathbb{R}^4$ denotes an uncertain function that maps $\theta(t)$ to the measurable[3] unit quaternion and is denoted by $q(t) \in \mathbb{R}^4$. The unit quaternion vector, denoted by $q(t) = [q_o(t),\, q_v^T(t)]^T$, with $q_o(t) \in \mathbb{R}$ and $q_v(t) \in \mathbb{R}^3$ [20, 21], provides a global, non-singular parametrization of the end-effector orientation and is subject to the constraint $q^T q = 1$. Several algorithms exist to determine the orientation of $\mathcal{E}$ relative to $\mathcal{B}$ from a rotation matrix that is a function of $\theta(t)$. Conversely, a rotation matrix, denoted by $R(q) \in SO(3)$, can be determined from a given $q(t)$ by the formula [4]:

$$R(q) = (q_o^2 - q_v^T q_v)I_3 + 2q_v q_v^T + 2q_o q_v^\times \qquad (7)$$

where I_3 is the 3×3 identity matrix and the notation $a^\times$, $\forall a = [a_1,\, a_2,\, a_3]^T$ denotes the following skew-symmetric matrix:

$$a^\times \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \qquad (8)$$

[2] The task-space position of $\mathcal{E}$ relative to $\mathcal{B}$ is assumed to be measurable, as in [7–18]. For example, a camera system or laser tracking could be utilized.

[3] The task-space orientation of $\mathcal{E}$ relative to $\mathcal{B}$ is also assumed to be measurable through the use of a camera system.

The time derivative of (6) is given by the following expression[4]:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} J_p \\ J_q \end{bmatrix} \dot{\theta} \tag{9}$$

where $J_p(\theta) : \mathbb{R}^6 \to \mathbb{R}^{3\times 6}$ and $J_q(\theta) : \mathbb{R}^6 \to \mathbb{R}^{4\times 6}$ denote the uncertain position and orientation Jacobian matrices, respectively, defined as $J_p(\theta) = \partial h_p/\partial\theta$ and $J_q(\theta) = \partial h_q/\partial\theta$. To facilitate the subsequent development, (9) is expressed as follows:

$$\begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(\theta)\dot{\theta} \text{ where } J(\theta) = \begin{bmatrix} J_p \\ B^T J_q \end{bmatrix} \in \mathbb{R}^{6\times 6} \tag{10}$$

The expression in (10) is obtained by exploiting the fact that $q(t)$ is related to the angular velocity of the end-effector, denoted by $\omega(t) \in \mathbb{R}^3$, via the following differential equation:

$$\omega = B^T \dot{q} \tag{11}$$

where the known Jacobian-like matrix, $B(q) : \mathbb{R}^4 \to \mathbb{R}^{4\times 3}$, is defined as follows:

$$B = \frac{1}{2} \begin{bmatrix} -q_v^T \\ q_o I_3 - q_v^\times \end{bmatrix} \tag{12}$$

Remark 1 *The dynamic and kinematic terms for a general revolute robot manipulator, denoted by $M(\theta)$, $V_m(\theta, \dot{\theta})$, $G(\theta)$, and $J(\theta)$, are assumed to depend on $\theta(t)$ only as arguments of trigonometric functions and, hence, remain bounded for all possible $\theta(t)$. During the control development, the assumption will be made that, if $p(t) \in \mathcal{L}_\infty$, then $\theta(t) \in \mathcal{L}_\infty$. (Note that $q(t)$ is always bounded, as $q^T q = 1$).*

Property 6 *The kinematic system in (10) can be linearly parametrized as follows:*

$$J\dot{\theta} = W_j \phi_j \tag{13}$$

where $W_j(\theta, \dot{\theta}) \in \mathbb{R}^{6\times n_1}$ denotes a regression matrix, which consists of known and measurable signals, and $\phi_j \in \mathbb{R}^{n_1}$ denotes a vector of n_1 unknown constants.

Property 7 *There exist upper and lower bounds for the parameter ϕ_j, such that $J(\theta, \phi_j)$ is always invertible. We will assume that the bounds for each parameter can be calculated as follows:*

$$\underline{\phi}_{ji} \le \phi_{ji} \le \overline{\phi}_{ji} \tag{14}$$

[4] To simplify the notation, the arguments of some functions in the equations are omitted. However, all functions are explicitly defined in the text.

where $\phi_{ji} \in \mathbb{R}$ denotes the ith component of $\phi_j \in \mathbb{R}^{n_1}$ and $\underline{\phi}_{ji}, \overline{\phi}_{ji} \in \mathbb{R}$ denotes the ith components of $\underline{\phi}_j, \overline{\phi}_j \in \mathbb{R}^{n_1}$, which are defined as follows:

$$\begin{aligned} \underline{\phi}_j &= [\underline{\phi}_{j1}, \underline{\phi}_{j2}, \ldots, \underline{\phi}_{jn_1}]^T \\ \overline{\phi}_j &= [\overline{\phi}_{j1}, \overline{\phi}_{j2}, \ldots, \overline{\phi}_{jn_1}]^T \end{aligned} \tag{15}$$

3. Problem Statement

The objective is to design the control input, $\tau(t)$, to ensure end-effector position and orientation tracking for the robot model given by (1) and (10), despite any parametric uncertainty in the kinematic and dynamic models. We will assume that the only measurable signals are the joint position, joint velocity, and end-effector position. To mathematically quantify this objective, the desired position and orientation of the robot end-effector is defined by a desired orthogonal coordinate frame, $\mathcal{E}_d$. The vector $p_d(t) \in \mathbb{R}^3$ denotes the position of the origin of $\mathcal{E}_d$, relative to the origin of $\mathcal{B}$, while the rotation matrix from $\mathcal{E}_d$ to $\mathcal{B}$ is denoted by $R_d(t) \in SO(3)$.

The end-effector position tracking error, $e_p(t) \in \mathbb{R}^3$, is defined as:

$$e_p = p_d - p \tag{16}$$

where $p_d(t)$, $\dot{p}_d(t)$, and $\ddot{p}_d(t)$ are assumed to be known, bounded functions of time. The orientation of $\mathcal{E}_d$, relative to $\mathcal{B}$, is specified in terms of a desired unit quaternion, $q_d(t) = [q_{od}(t), q_{vd}^T(t)]^T \in \mathbb{R}^4$, with $q_{od}(t) \in \mathbb{R}$ and $q_{vd}(t) \in \mathbb{R}^3$. Then, as with (7), the rotation matrix from $\mathcal{E}_d$ to $\mathcal{B}$ can be calculated from the desired unit quaternion, $q_d(t)$, as follows:

$$R_d(q_d) = (q_{od}^2 - q_{vd}^T q_{vd}) I_3 + 2 q_{vd} q_{vd}^T + 2 q_{od} q_{vd}^\times \tag{17}$$

where it is assumed that $R_d, \dot{R}_d, \ddot{R}_d \in \mathcal{L}_\infty$. As in (11), the time derivative of $q_d(t)$ is related to the desired angular velocity of the end-effector (the angular velocity of $\mathcal{E}_d$ relative to $\mathcal{B}$), denoted by $\omega_d(t) \in \mathbb{R}^3$, through the known kinematic equation:

$$\dot{q}_d = B(q_d)\omega_d \tag{18}$$

To quantify the difference between the actual and desired end-effector orientations, we define the rotation matrix $\tilde{R} \in SO(3)$, from $\mathcal{E}$ to $\mathcal{E}_d$, as follows:

$$\tilde{R} \triangleq R_d^T R = (e_o^2 - e_v^T e_v) I_3 + 2 e_v e_v^T + 2 e_o e_v^\times \tag{19}$$

where $e_q(t) \triangleq [e_o(t), e_v^T(t)]^T \in \mathbb{R}^4$ represents the unit quaternion tracking error that satisfies the constraint:

$$e_q^T e_q = e_o^2 + e_v^T e_v = 1 \tag{20}$$

The quaternion tracking error, $e_q(t)$, can be explicitly calculated from $q(t)$ and $q_d(t)$, via quaternion algebra, by noticing that the quaternion equivalent of $\tilde{R} = R_d^T R$ is the following quaternion product [6, 21]:

$$e_q = q q_d^* \tag{21}$$

where $q_d^*(t) \triangleq [q_{od}(t),\ -q_{vd}^T(t)]^T \in \mathbb{R}^4$ is the unit quaternion representing the rotation matrix, $R_d^T(q_d)$. After using quaternion algebra, the quaternion tracking error can be derived as follows (see [6] and Theorem 5.3 of [21]):

$$\begin{bmatrix} e_o \\ e_v \end{bmatrix} = \begin{bmatrix} q_o q_{od} + q_v^T q_{vd} \\ q_{od} q_v - q_o q_{vd} + q_v^\times q_{vd} \end{bmatrix} \tag{22}$$

Based on (11), (18), and (22), the unit quaternion error system can be formulated as follows [22]:

$$\begin{bmatrix} \dot{e}_o \\ \dot{e}_v \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} e_v^T \tilde{\omega} \\ \frac{1}{2}(e_o I_3 - e_v^\times)\tilde{\omega} \end{bmatrix} \tag{23}$$

The angular velocity of $\mathcal{E}$ with respect to $\mathcal{E}_d$, with coordinates in $\mathcal{E}_d$, denoted by $\tilde{\omega}(t) \in \mathbb{R}^3$, can be calculated from (19) as follows [23]:

$$\tilde{\omega} = R_d^T(\omega - \omega_d) \tag{24}$$

The end-effector tracking errors are then written using (10), (16), and (24) as:

$$\begin{bmatrix} \dot{e}_p \\ \tilde{\omega} \end{bmatrix} = \Lambda \left(\begin{bmatrix} -\dot{p}_d \\ -\omega_d \end{bmatrix} + J\dot{\theta} \right) \tag{25}$$

where $\Lambda \in \mathbb{R}^{6\times 6}$ is defined as:

$$\Lambda = \begin{bmatrix} -I_3 & 0_{3\times 3} \\ 0_{3\times 3} & R_d^T \end{bmatrix} \tag{26}$$

where $0_{3\times 3}$ represents a 3×3 matrix of zeros. Based on the above definitions, the tracking objective, defined in terms of the end-effector position and unit quaternion error, is to design the control input, $\tau(t)$, such that:

$$\|e_p(t)\| \to 0 \quad \text{and} \quad \|e_v(t)\| \to 0 \quad \text{as} \quad t \to \infty \tag{27}$$

The orientation tracking objective given in (27) can also be stated in terms of $e_q(t)$. Specifically, (20) implies that:

$$0 \le \|e_v(t)\| \le 1 \qquad \text{and} \qquad 0 \le |e_o(t)| \le 1 \tag{28}$$

for all time and, if $\|e_v(t)\| \to 0$ as $t \to \infty$, then $e_o(t) \to 1$ as $t \to \infty$. Thus, if $\|e_v(t)\| \to 0$ as $t \to \infty$, then (19), along with the previous statement, can be used to conclude that $\tilde{R}(t) \to I_3$ as $t \to \infty$. Hence, the orientation tracking objective can be achieved.

4. Tracking Error System Development

To facilitate the development of the open-loop error system, an auxiliary variable, $\eta(t) \in \mathbb{R}^6$, is defined as follows:

$$\eta = (\Lambda\hat{J})^{-1} \begin{bmatrix} \dot{p}_d + k_1 e_p \\ -R_d^T \omega_d + k_2 e_v \end{bmatrix} + \dot{\theta} \tag{29}$$

where $k_1, k_2 \in \mathbb{R}^{3\times 3}$ are positive, constant, diagonal matrices and $\hat{J}(\theta, \hat{\phi}_j) \in \mathbb{R}^{6\times 6}$ is an estimated manipulator Jacobian matrix. After adding and subtracting the terms $\Lambda\hat{J}(\theta, \hat{\phi}_j)\dot{\theta}(t)$ and $\Lambda\hat{J}(\theta, \hat{\phi}_j)\eta(t)$ to (25), and utilizing (26), the following kinematic error system can be developed:

$$\begin{bmatrix} \dot{e}_p \\ \tilde{\omega} \end{bmatrix} = -\begin{bmatrix} k_1 e_p \\ k_2 e_v \end{bmatrix} + \Lambda(\hat{J}\eta + W_j\tilde{\phi}_j) \tag{30}$$

where $W_j(\cdot) \in \mathbb{R}^{6\times n_1}$, which was introduced in (13), and the parameter estimation error term $\tilde{\phi}_j(t) \in \mathbb{R}^{n_1}$ is defined as:

$$\tilde{\phi}_j = \phi_j - \hat{\phi}_j \tag{31}$$

The adaptive estimate $\hat{\phi}_j(t) \in \mathbb{R}^{n_1}$, introduced in (31), is designed as follows:

$$\dot{\hat{\phi}}_j = proj\{y\} \tag{32}$$

where the auxiliary term, $y \in \mathbb{R}^{n_1}$, is defined as:

$$y = \Gamma_1 W_j^T \Lambda^T \begin{bmatrix} e_p \\ e_v \end{bmatrix} \tag{33}$$

where $\Gamma_1 \in \mathbb{R}^{n_1 \times n_1}$ is a constant positive diagonal matrix and the function $proj\{y\}$ is defined as:

$$proj\{y_i\} \triangleq \begin{cases} y_i & \text{if } \hat{\phi}_{ji} > \underline{\phi}_{ji} \\ y_i & \text{if } \hat{\phi}_{ji} = \underline{\phi}_{ji} \quad \text{and } y_i > 0 \\ 0 & \text{if } \hat{\phi}_{ji} = \underline{\phi}_{ji} \quad \text{and } y_i < 0 \\ 0 & \text{if } \hat{\phi}_{ji} = \overline{\phi}_{ji} \quad \text{and } y_i > 0 \\ y_i & \text{if } \hat{\phi}_{ji} = \overline{\phi}_{ji} \quad \text{and } y_i \le 0 \\ y_i & \text{if } \hat{\phi}_{ji} < \overline{\phi}_{ji} \end{cases} \tag{34}$$

$$\underline{\phi}_{ji} \le \hat{\phi}_{ji}(0) \le \overline{\phi}_{ji} \tag{35}$$

where y_i denotes the ith component of y and $\hat{\phi}_{ji}(t)$ denotes the ith component of $\hat{\phi}_j(t)$. (Note that the above projection algorithm ensures that $\underline{\phi}_j \le \hat{\phi}_j(t) \le \overline{\phi}_j$ and, hence, using Property 7, we can observe that the estimated manipulator Jacobian matrix, $\hat{J}(\theta, \hat{\phi}_j)$, will always be non-singular. For further details of the projection algorithm the reader is referred [24]).

To obtain the closed loop error system for $\eta(t)$, we first take the time derivative of (29) to obtain the following expression:

$$\dot{\eta} = \frac{d}{dt}\left\{(\Lambda\hat{J})^{-1}\begin{bmatrix}\dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v\end{bmatrix}\right\} + \ddot{\theta} \quad (36)$$

After pre-multiplying (36) by $M(\theta)$, substituting (1) into the resulting expression for $M(\theta)\ddot{\theta}(t)$, and utilizing (29), the following simplified expression can be obtained:

$$M\dot{\eta} = -V_m\eta + \tau + W_y\phi_y \quad (37)$$

where $W_y(p_d, \dot{p}_d, \ddot{p}_d, q_d, \omega_d, \dot{\omega}_d, p, q, \theta, \dot{\theta}) \in \mathbb{R}^{6\times n_2}$ is a regression matrix of known and measurable quantities and $\phi_y \in \mathbb{R}^{n_2}$ is a vector of n_2 unknown constant parameters. The product $W_y(\cdot)\phi_y$, introduced in (37), is defined as:

$$\begin{aligned} W_y\phi_y = {} & M\frac{d}{dt}\left\{(\Lambda\hat{J})^{-1}\begin{bmatrix}\dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v\end{bmatrix}\right\} \\ & + V_m(\Lambda\hat{J})^{-1}\begin{bmatrix}\dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v\end{bmatrix} \\ & - G(\theta) - F_d\dot{\theta} \end{aligned} \quad (38)$$

Based on (37), and the subsequent stability analysis, the control input, $\tau(t)$, is designed as:

$$\tau = -W_y\hat{\phi}_y - k_r\eta - (\Lambda\hat{J})^T\begin{bmatrix}e_p \\ e_v\end{bmatrix} \quad (39)$$

where $k_r \in \mathbb{R}^{6\times 6}$ is a constant, positive, diagonal matrix and $\hat{\phi}_y(t) \in \mathbb{R}^{n_2}$ denotes an adaptive estimate, which is generated by the following differential expression:

$$\dot{\hat{\phi}}_y = \Gamma_2 W_y^T\eta \quad (40)$$

where $\Gamma_2 \in \mathbb{R}^{n_2\times n_2}$ is a positive, constant, diagonal matrix. After substituting (39) into (37), the following closed-loop error system is obtained:

$$M\dot{\eta} = -V_m\eta + W_y\tilde{\phi}_y - k_r\eta - (\Lambda\hat{J})^T\begin{bmatrix}e_p \\ e_v\end{bmatrix} \quad (41)$$

where the adaptive estimation error is defined as:

$$\tilde{\phi}_y = \phi_y - \hat{\phi}_y \quad (42)$$

Remark 2 *Based on the definition of the quaternion error system in (23), the kinematic error system in (30), and the regression matrix in (38), we can conclude that $W_y(\cdot)$ does not require the measurement of the task-space velocity. Further, from the definition of $e_p(t), e_v(t)$, and $\eta(t)$, it is clear that the control input torque, $\tau(t)$, does not require measurement of the task space velocity.*

5. Stability Analysis

Theorem 1 *Given the robotic system described by (1), the control input (39), along with the adaptive laws defined in (32) and (40), guarantee asymptotic regulation of the end-effector position error and the unit quaternion error, in the sense that $\|e_p(t)\| \to 0$ as $t \to \infty$ and $\|e_v(t)\| \to 0$ as $t \to \infty$, thus completing the position and orientation tracking objective.*

Proof Let $V(t) \in \mathbb{R}$ denote the following non-negative scalar function:

$$\begin{aligned} V = {} & \frac{1}{2}e_p^T e_p + (1 - e_0)^2 + e_v^T e_v + \frac{1}{2}\eta^T M\eta \\ & + \frac{1}{2}\tilde{\phi}_j^T\Gamma_1^{-1}\tilde{\phi}_j + \frac{1}{2}\tilde{\phi}_y^T\Gamma_2^{-1}\tilde{\phi}_y \end{aligned} \quad (43)$$

After taking the time derivative of (43) and utilizing (23), (31), and (42), the following expression is obtained:

$$\begin{aligned} \dot{V} = {} & e_p^T\dot{e}_p + (1 - e_0)(e_v^T\tilde{\omega}) + e_v^T(e_0 I_3 - e_v^\times)\tilde{\omega} \\ & + \frac{1}{2}\eta^T\dot{M}\eta + \eta^T M\dot{\eta} - \tilde{\phi}_y^T\Gamma_2^{-1}\dot{\hat{\phi}}_y - \tilde{\phi}_j^T\Gamma_1^{-1}\dot{\hat{\phi}}_j \end{aligned} \quad (44)$$

Upon further simplification of equation (44), by cancelling common terms, and substituting for $M\dot{\eta}$ from (41), the following expression for $\dot{V}(t)$ can be obtained:

$$\begin{aligned} \dot{V} = {} & \begin{bmatrix}e_p^T & e_v^T\end{bmatrix}\begin{bmatrix}\dot{e}_p \\ \tilde{\omega}\end{bmatrix} - \eta^T k_r\eta - \eta^T V_m\eta \\ & + \eta^T W_y\tilde{\phi}_y - \eta^T(\Lambda\hat{J})^T\begin{bmatrix}e_p \\ e_v\end{bmatrix} \\ & + \frac{1}{2}\eta^T\dot{M}\eta - \tilde{\phi}_j^T\Gamma_1^{-1}\dot{\hat{\phi}}_j - \tilde{\phi}_y^T\Gamma_2^{-1}\dot{\hat{\phi}}_y \end{aligned} \quad (45)$$

After using Property 3, substituting from (30), (32), and (40), and cancelling terms, $\dot{V}(t)$ can be expressed as:

$$\begin{aligned} \dot{V} = {} & [\,e_p^T\ e_v^T\,]\left(-\begin{bmatrix}k_1 e_p \\ k_2 e_v\end{bmatrix} + \Lambda W_j\tilde{\phi}_j\right) \\ & - \eta^T k_r\eta - \tilde{\phi}_j^T\Gamma_1^{-1}proj\{y\} \end{aligned} \quad (46)$$

Substituting for y from (33) and using the definition of the projection function, (34), the expression for $\dot{V}(t)$ can be upper bounded as follows:

$$\begin{aligned} \dot{V} \le {} & -\lambda_{min}\{k_1\}\|e_p\|^2 - \lambda_{min}\{k_2\}\|e_v\|^2 \\ & -\lambda_{min}\{k_r\}\|\eta\|^2 \end{aligned} \quad (47)$$

where λ_{min} is the minimum Eigenvalue of the matrix.

The expressions in (43) and (47) can be used to prove that $e_p(t)$, $e_v(t)$, $\eta(t)$, $\tilde{\phi}_j(t)$, $\tilde{\phi}_y(t) \in \mathcal{L}_\infty$ and that $e_p(t)$, $e_v(t)$, $\eta(t) \in \mathcal{L}_2$. Using (16), and the assumption that $p_d(t) \in \mathcal{L}_\infty$, it is clear that $p(t) \in \mathcal{L}_\infty$. From (31) and (42) it can be concluded that $\hat{\phi}_j(t)$, $\hat{\phi}_y(t) \in \mathcal{L}_\infty$. Utilizing Property 7, the definition of $\eta(t)$ in (29), and the fact that $e_p(t)$, $e_v(t)$, $\eta(t) \in \mathcal{L}_\infty$, we can show that $\dot{\theta}(t) \in \mathcal{L}_\infty$. Moreover, (9), (16), and the fact that $J(\theta) \in \mathcal{L}_\infty$, can be used to show that $\dot{p}(t)$, $\dot{e}_p(t) \in \mathcal{L}_\infty$. From (20), (23), (25), and (28) we can show that $e_0(t)$, $\dot{e}_0(t)$, $\dot{e}_v(t) \in \mathcal{L}_\infty$. From the definition of $W_j(\cdot)$ and $W_y(\cdot)$ in (13) and (38), respectively, and the preceding arguments, it is clear that $W_y(\cdot)$, $W_j(\cdot) \in \mathcal{L}_\infty$. Utilizing (32), (33), (34), and (40), we can show that $\dot{\hat{\phi}}_j(t)$, $\dot{\hat{\phi}}_y(t) \in \mathcal{L}_\infty$. The definition of $\tau(t)$ in (39) can be used to show that $\tau(t) \in \mathcal{L}_\infty$. Hence, $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t) \in \mathcal{L}_\infty$. From (36) we can conclude that $\dot{\eta}(t) \in \mathcal{L}_\infty$. As $\dot{e}_p(t)$, $\dot{e}_v(t)$, $\dot{\eta}(t) \in \mathcal{L}_\infty$ and $e_p(t)$, $e_v(t)$, $\eta(t) \in \mathcal{L}_2$, Barbalat's Lemma [25] can be used to show that $\|e_p(t)\| \to 0$, $\|e_v(t)\| \to 0$ and $\|\eta(t)\| \to 0$ as $t \to \infty$.

6. Experimental Results

The developed controller was implemented on the Barrett whole arm manipulator (WAM). The WAM is a seven degrees of freedom (d.o.f.), highly dexterous, and backdrivable robotic manipulator. The objective of the experiment is to verify the performance of the developed adaptive controller. So, to simplify the controller implementation, five joints of the robot were locked at fixed angles and the remaining links of the manipulator were used as a two d.o.f. planar robot manipulator (refer to Fig. 1). The dynamics of the robot in this planar configuration can be expressed as [23]:

$$\tau = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} V_{m11} & V_{m12} \\ V_{m21} & V_{m22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_1 \end{bmatrix} \tag{48}$$

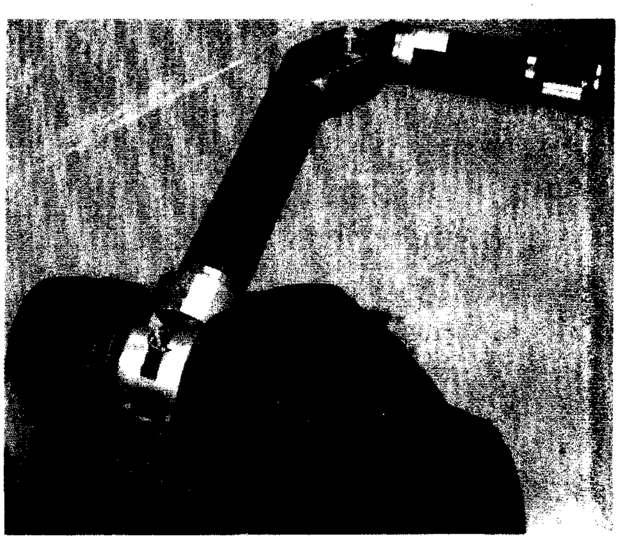

Figure 1. Barrett whole arm manipulator.

The elements of the inertia and centripetal-Coriolis matrices are defined as follows:

$$\begin{aligned} M_{11} &= m_1 l_{c1}^2 + m_2 l_{c2}^2 + m_2 l_1^2 + 2m_2 l_1 l_{c2}\cos(\theta_2) \\ M_{12} &= m_2 l_{c2}^2 + m_2 l_1 l_{c2}\cos(\theta_2) \\ M_{21} &= M_{12} \quad M_{22} = m_2 l_{c2}^2 \\ V_{m11} &= -m_2 l_1 l_{c2}\sin(\theta_2)\dot{\theta}_2 \\ V_{m12} &= -m_2 l_1 l_{c2}\sin(\theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \\ V_{m21} &= m_2 l_1 l_{c2}\sin(\theta_2)\dot{\theta}_1 \quad V_{m22} = 0 \end{aligned} \tag{49}$$

where m_1, $m_2 \in \mathbb{R}$ denotes the mass of the links, l_1, $l_2 \in \mathbb{R}$ denotes the length of the links, and l_{c1}, $l_{c2} \in \mathbb{R}$ denotes the distance to the centre of mass. The terms f_{d1}, $f_{d2} \in \mathbb{R}$ in (48) denote the uncertain friction coefficients of the manipulator. The vector of uncertain constant dynamic parameters $\phi_y \in \mathbb{R}^{14}$ was found to be:

$$\phi_y = [\phi_{y1}, \ldots, \phi_{y14}]^T \tag{50}$$

where

$$\begin{aligned} &\phi_{y1} = m_1 l_1 l_{c1}^2, && \phi_{y2} = m_1 l_2 l_{c1}^2, && \phi_{y3} = m_1 l_{c1}^2, \\ &\phi_{y4} = m_2 l_1 l_{c2}^2, && \phi_{y5} = m_2 l_2 l_{c2}^2, && \phi_{y6} = m_2 l_{c2}^2, \\ &\phi_{y7} = m_2 l_1^3, && \phi_{y8} = m_2 l_1^2, && \phi_{y9} = m_2 l_1^2 l_{c2}, \\ &\phi_{y10} = m_2 l_1 l_2 l_{c2}, && \phi_{y11} = m_2 l_1^2 l_2, && \phi_{y12} = m_2 l_1 l_{c2}, \\ &\phi_{y13} = f_{d1}, && \phi_{y14} = f_{d2}. \end{aligned}$$

The control algorithm was written in "C++" and hosted on an AMD Athlon 1.2 GHz PC operating under QNX 6.2.1. Data logging and on-line gain tuning were performed using Qmotor 3.0 control software [26]. Data acquisition and control implementation were performed at a frequency of 1.0 kHz, using the ServoToGo I/O board. Joint positions were measured using the optical encoders located at the motor shaft of each axis. Joint velocity measurements were obtained using a filtered, backwards difference algorithm.

Remark 3 *The kinematics of the robotic system are assumed to be unknown. The task-space variable is assumed to be measured using an external sensor (a camera system or laser tracking could be used). To simplify the experiment, the task-space measurements were simulated by using the known kinematics of the robot (that is, we artificially generated the task-space position measurements using the known forward kinematics). This kinematic information was used only to artificially generate the task-space signals and was not used to generate any other signals in the control algorithm.*

The approximated Jacobian matrix, used in the control implementation, is defined as follows:

$$\hat{J}_p = \begin{bmatrix} -\hat{l}_1 S_1 - \hat{l}_2 S_{12} & -\hat{l}_2 S_{12} \\ \hat{l}_1 C_1 + \hat{l}_2 C_{12} & \hat{l}_2 C_{12} \end{bmatrix} \tag{51}$$

where $\hat{J}_p \in \mathbb{R}^{2\times 2}$, $\hat{l}_1$, and $\hat{l}_2$ are estimates for the link lengths $S_1 = \sin(\theta_1)$, $S_{12} = \sin(\theta_1 + \theta_2)$, $C_1 = \cos(\theta_1)$, and $C_{12} = \cos(\theta_1 + \theta_2)$. The parameter vector, $\hat{\phi}_j \in \mathbb{R}^2$, is defined as:

$$\hat{\phi}_j = [\hat{l}_1 \hat{l}_2]^T \tag{52}$$

and the estimates were initialized to $\hat{l}_1(0) = 0.42$ [m] and $\hat{l}_2(0) = 0.22$ [m].

The true link lengths are $l_1 = 0.558$ [m] and $l_2 = 0.291$ [m]. We initialized the link length estimates to 75 percent of the true value. In cases where there was no information available about the link lengths, a best-guess could be used as an initial estimate of the link lengths.

A circular desired trajectory was selected for the end-effector, which was defined as follows:

$$p_d = \begin{bmatrix} 0.55 + 0.2\cos(2t) \\ 0.25 + 0.2\sin(2t) \end{bmatrix} \tag{53}$$

The initial position of the joints were $\theta_1(0) = 3.3°$ and $\theta_2(0) = 45.1°$, which corresponds to $x(0) = 0.75$ [m], $y(0) = 0.25$ [m] in the task-space. The control gains that yielded the best tracking performance were as follows:

$$\begin{aligned} k_1 &= diag\{2.5, 2.0\}, k_r = diag\{80, 40\}, \Gamma_1 = diag\{8, 1\} \\ \Gamma_2 &= diag\{20, 45, 10, 500, 1, 3, 8, 15, 20, 5, 500, 25, 20, 20\} \end{aligned} \tag{54}$$

Remark 4 *In this planar two degree of freedom example, there was no rotational error,* $e_v(t)$*; as such the gain* k_2 *is not used.*

Fig. 2 shows the actual and desired circular end-effector trajectories for the last revolution. Fig. 3 shows the end-effector position tracking error. It shows that, within approximately 10 seconds, the tracking error converged to ± 2 [mm]. From Fig. 2 and Fig. 3 it can clearly be seen that the position tracking error for the end-effector is very small. Fig. 4 shows the kinematic parameter estimates, that is, the link length estimates $\hat{\phi}_j(t)$ defined in (52). It is interesting to note that, although there is no guarantee of the parameter estimates convergence to their true values, the link length estimates shown in Fig. 4 quite closely approach their true values. Fig. 5 shows the dynamic parameter estimates: the estimates, $\phi_y(t)$, for the uncertain dynamic parameters defined in (50). From Figs. 3, 4, and 5, it is clear that the tracking error converges as the kinematic and dynamic parameters converge. Fig. 6 shows the control input torques to the two links of the Barrett WAM.

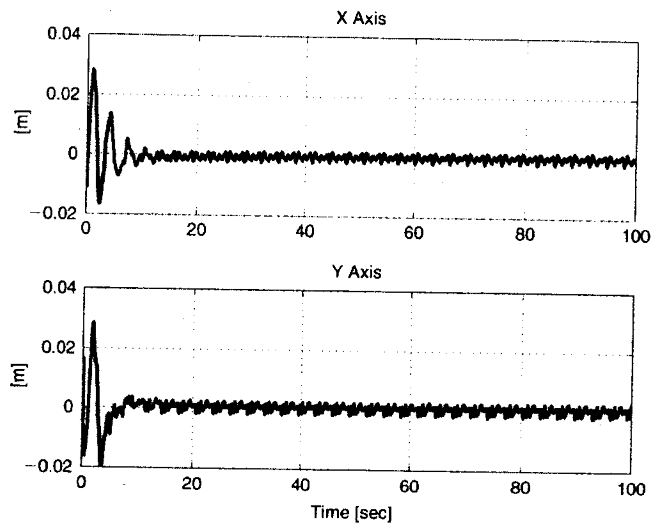

Figure 3. End-effector position tracking error.

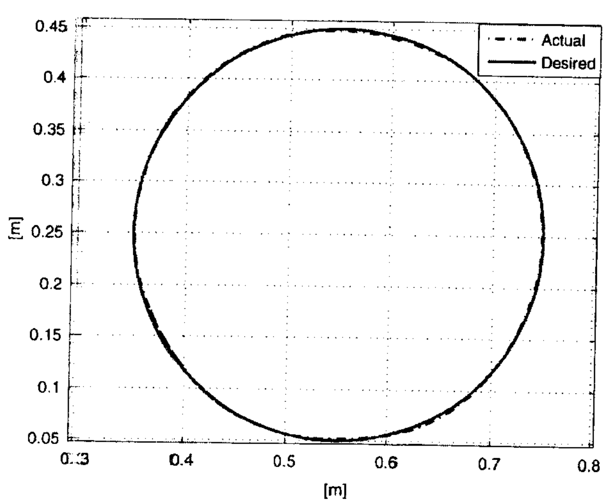

Figure 2. Actual and desired end-effector trajectory (only the last revolution is shown).

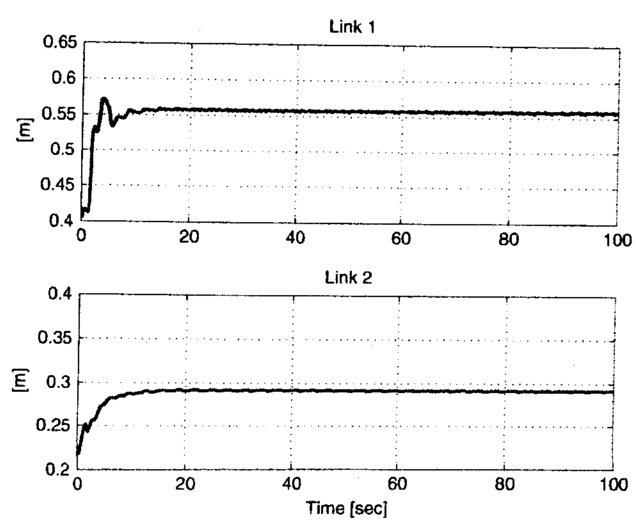

Figure 4. Estimates of the link lengths.

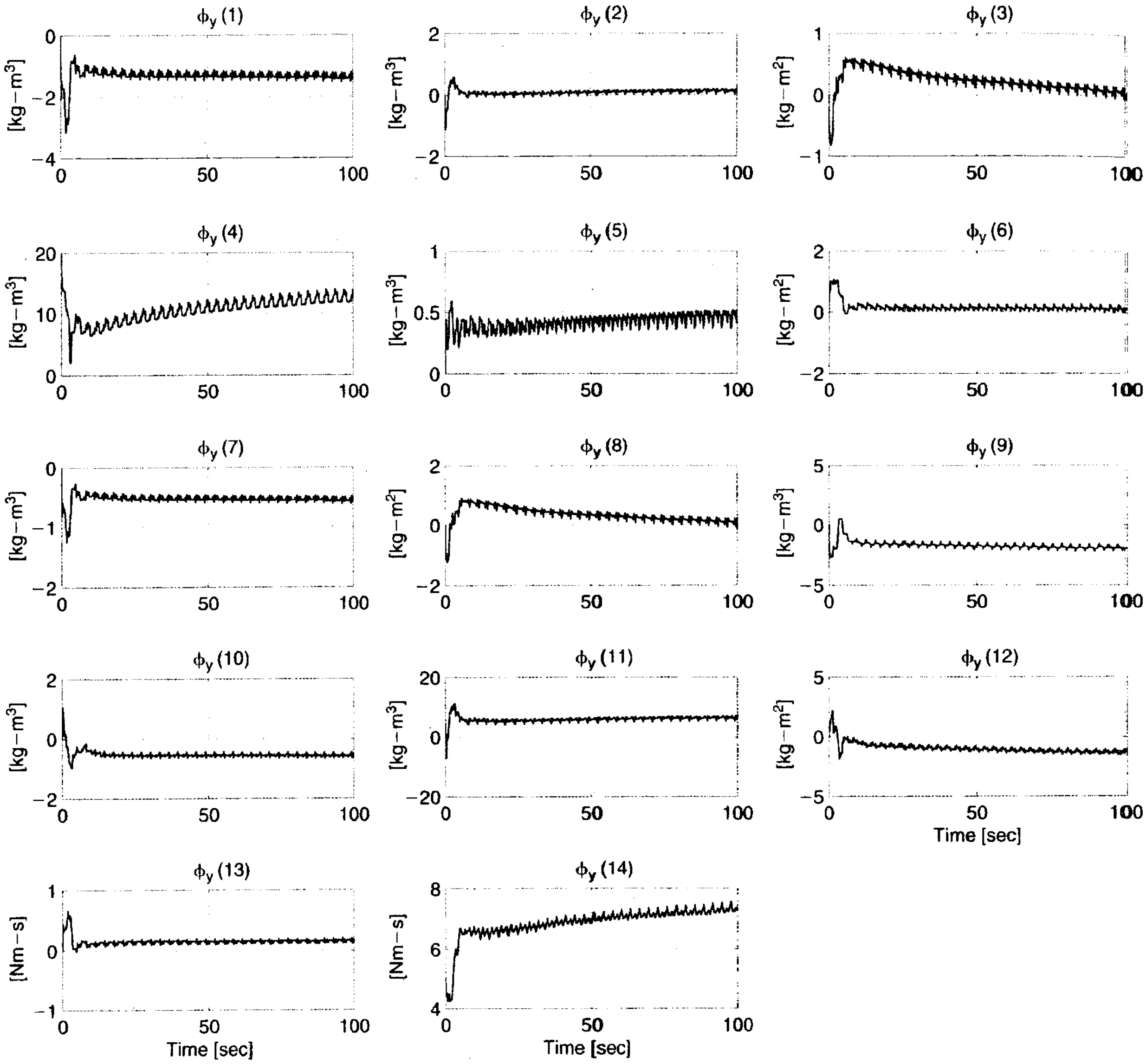

Figure 5. Estimates of the uncertain dynamic parameters defined in (50).

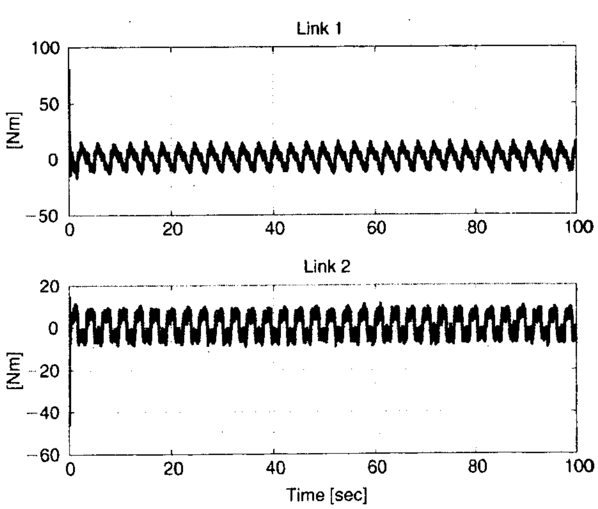

Figure 6. Control input torques.

7. Conclusion

A task-space, adaptive tracking controller for robot manipulators, with uncertainty in both the kinematic and the dynamic models, was proposed. The controller yields asymptotic regulation of the end-effector position and orientation tracking errors. The advantages of the proposed controller are that singularities associated with the three parameter representation are avoided and, unlike with previous work in this area, the controller does not require the measurement of the task-space velocity. The experiment carried out on a planar, two link configuration of the Barrett WAM validates the performance of the proposed controller.

Acknowledgements

This work is supported in part by a DOC Grant, an ARO Automotive Center Grant, a DOE Contract, a Honda

Corporation Grant, and a DARPA Contract at Clemson University. It was also supported by the NSF CAREER award CMS-0547448, AFOSR contract numbers F49620-03-1-0381 and F49620-03-1-0170, AFRL contract number FA4819-05-D-0011, and by research grant No. US-3715-05 from BARD, the United States – Israel Binational Agricultural Research and Development Fund at the University of Florida.

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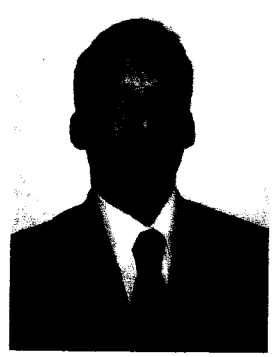
Biographies



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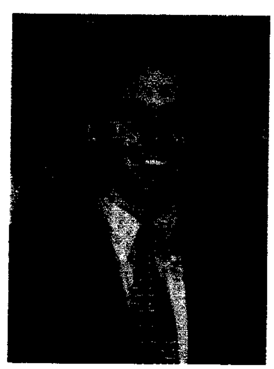

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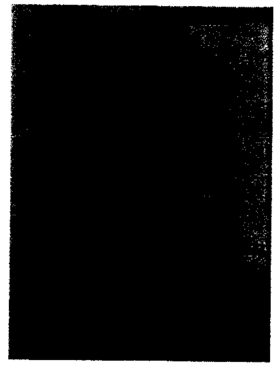

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