Indirect Herding Control Through a Chain-of-Influence by Uncertain Intermediaries

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***Abstract*—Herding is an established class of problems where a herding agent seeks to influence a target agent to move to a desired location. Motivated by scenarios where the herder cannot directly influence a target or the herder desires to conceal its influence, this letter examines a new generalization of the herding problem where the herding agent influences a target agent through a set of intermediate agents (i.e., multi-hop chain of influence). The interconnected influences result in challenging coupled dynamics within the group of agents. Motivated by this challenge, robust control techniques are used to compensate for the complex chain of influences among the group, and a Lyapunov-based analysis is used to prove the target is exponentially regulated to a desired location by the herding agent.**

***Index Terms*—Indirect regulation, robust control, nonlinear systems.**

I. Introduction

HERDING (also called indirect control) is an established class of control problems where a herding agent seeks to influence a target agent to a desired location. Inspired by shepherding in nature, a herding agent exploits an interaction dynamic to indirectly guide sheep to a goal location. The herding problem has applications in social network influence [1], [2] and in wildlife management, where animals may need to be deterred or guided to prevent harm to themselves or people [3]. In works such as [4], an uncrewed aerial vehicle was programmed with a path-planning herding algorithm to guide flocks of birds away from a protected zone. Applications of related problems such as pursuit-evasion and target tracking are prevalent in defense [5], [6], [7], [8] and environmental conservation [9].

Indirect control evaluates various multi-agent combinations involving herding and target agents [4], [10], [11], [12], [13], [14], [15], [16]. Some approaches use an enclosure method with multiple herding agents to prevent target agents from escaping while being influenced [11], [14], [15], [17]. Specific examples of this include StringNet, a sequence of capture and influence [14], caging [15], [17], and circular formations that can be open or closed [11], [18]. When multiple herding agents are not available, other strategies have been developed [12], [19], [20], [21], [22]. In [12], [20], [21], a switching strategy was developed based on a dwell time analysis to provide a minimum time of engagement for the herding agent to influence a target agent to prevent it from fleeing. In contrast [22], used a technique of circling around the group of target agents to eliminate the need of switching; however, the result is limited to specific initial conditions for the target agents. A collecting and driving method, uses a herding agent to drive target agents toward a goal and then switch to perpendicular collecting movements for containment [19]. The herding problem increases in difficulty in the presence of unknown environmental factors or when enclosing the target agents is not feasible.

None of the aforementioned results analyze a group of agents that interact through leveraging a sequence of influences. The closest research that addresses the herding problem for a model of this form are those in [4], [10], [19], where agents are studied via their collective center-of-mass, which incorporates interactions between the non-cooperative agents. The center-of-mass approach does not directly focus on leveraging the inter-agent dynamics to complete the tracking goal, because it evaluates the entire error of the system based on all agents. Similarly, in [23] a technique of influencing the agent furthest from the goal location was formulated, but this also fails to leverage other existing agents since it focuses on a single agent. Motivated by scenarios where the herder can not directly influence a target agent or the herder desires to conceal its influence, this letter examines a new generalization of the herding problem where the herding agent influences a target agent by leveraging a set of intermediate agents. The chain of influence problem can be viewed as a deceptive or veiled form of herding where the goal of the directly controlled herding agent is not known or easily detectable by the intermediate agent or the target agent. Control laws that focus on the individual dynamics of agents that are being leveraged to influence other agents has not been evaluated. The concatenation of leveraging unknown interaction functions onto other unknown interaction functions can be evaluated as levels of influence, or "hops," that trickle down to the influence of the target agent.

The contribution of this letter is the first development where a herding agent indirectly influences a non-cooperative target agent to a goal location indirectly by influencing a series of non-cooperative intermediate agents, i.e., a multi-hop herding problem. An example is demonstrated with a group of four

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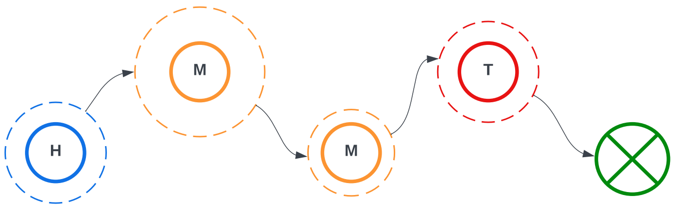

Fig. 1. Here, H denotes the herding agent, M denotes the mediator agents, T denotes the target agent, and the green circle denotes the goal location. The dashed circles of different radii around each agent represents influencing heterogeneity among agents, and the arrows represent the general idea of how a chain of inter-agent interactions can be constructed to influence the target agent to a goal location.

agents: one herding agent, two intermediate agents, and a target agent. As depicted in Figure 1, the group of agents operate through a sequence of influences, where each agent is influenced by its predecessor and influences the next agent along the chain. The first agent in the chain, the herder, is the only agent that can be directly controlled, and is the only agent that knows the desired end goal location of the target agent. Consequently, the herding agent must leverage the entire sequence of influences to indirectly guide the target agent, the last agent in the chain. The sequential structure of the interaction functions motivates the use of integrator backstepping; however, since the influence that each agent can exert on the next agent is unknown, a multiplicative uncertainty appears in each subsequent virtual control input. To address the challenge imposed by the multiplicative uncertainty, a robust control method is derived from a Lyapunov-based stability analysis to guarantee exponential target agent regulation to a goal location.

II. Problem Formulation

A. System Dynamics

A group of N agents is indexed by i, where $\mathcal{I} \triangleq \{1, \ldots, N\}$ denotes the set of influenced agents, $\mathcal{T} \triangleq \{N\}$ denotes the singleton set of the target agent, and $\mathcal{M} \triangleq \{1, \ldots, N-1\}$ denotes the set of mediator agents such that $\mathcal{I} = \mathcal{T} \cup \mathcal{M}$. Additionally, $\mathcal{H} \triangleq \{0\}$ denotes the singleton set of the herding agent and $\mathcal{C} = \mathcal{H} \cup \mathcal{M}$ represents the set of influence agents. The dynamics for agents $i \in \mathcal{I}$ are

$$\dot{x}_i = f_i(x_i, x_{i-1})(x_i - x_{i-1}), \tag{1}$$

where $x_i \in \mathbb{R}^n$ denotes the known state of the i^{th} agent, and $f_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{>0}$ denotes the unknown locally Lipschitz interaction function between agents x_i and x_{i-1}, respectively. The dynamics for the herding agent are

$$\dot{x}_0 = d(x_0, t) + u, \tag{2}$$

where $x_0 \in \mathbb{R}^n$ denotes the position of the herding agent, $d : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ denotes the unknown locally Lipschitz drift dynamics affecting the herding agent, and $u \in \mathbb{R}^n$ denotes the control input of the herding agent.[1] A visualization of the problem is shown in Figure 1. Only the herding agent has access to the position information of all agents and knows the goal location.

[1]More complex and uncertain herder dynamics can be considered; however the dynamics in (2) are considered for ease of exposition. The challenge in this problem results from the uncertainty in the influence functions in (1).

Assumption 1: There exist known constants $\underline{f_i}, \overline{f_i} \in \mathbb{R}_{>0}$ for the interaction function such that $\underline{f_i} \leq f_i(x_i, x_{i-1}) \leq \overline{f_i}$, $\forall i \in \mathcal{I}$ [12].

B. Control Objective

The objective is to regulate the target agent to a goal location $x_g \in \mathbb{R}^n$, known only to the herding agent x_0, despite the chain of unknown interaction dynamics in (1). To quantify the control objective, the regulation error $e_i \in \mathbb{R}^n$ for all $i \in \mathcal{H} \cup \mathcal{I}$ is defined as

$$e_i \triangleq x_i^d - x_i. \tag{3}$$

where $x_i^d \in \mathbb{R}^n$ represents a subsequently designed virtual desired trajectory for agent x_i to track. In (3), $x_i^d \triangleq x_g$ for $i \in \mathcal{T}$. Given that only the herding agent has an implementable control input, we design virtual desired trajectories x_i^d for the mediator agents to track to leverage a chain of inter-agent interactions. The virtual desired trajectories for $i \in \mathcal{C}$ are designed as

$$x_i^d \triangleq x_{i+1} - K_{i+1} e_{i+1}, \tag{4}$$

where $K_{i+1} \in \mathbb{R}_{>0}$ is a user-defined gain and x_{i+1} is the desired subsequent agent in the chain of influence to be tracked. The subsequent controller will have the form

$$u \triangleq K_0 e_0 + \sum_{i=1}^{N} \beta_i e_i + v_r, \tag{5}$$

where $e_0 \in \mathbb{R}^n$ is the herding agent's trajectory error, $K_0 \in \mathbb{R}_{>0}$ is a user-defined positive control gain, $\beta_i \in \mathbb{R}$ are bounding constants, and $v_r \in \mathbb{R}^n$ represents a compensatory term to mitigate the unknown drift dynamics.[2]

III. Control Design

A. Virtual Trajectory Design

Consider $N = 3$ agents.[3] To facilitate the control design, let $\kappa_i \triangleq [x_i^\top x_{i-1}^\top]^\top$ for all $i \in \mathcal{I}$. Adding and subtracting $f_i(\kappa_i) x_{i-1}^d$ to the right-hand side of (1), using (3), and simplifying yields

$$\dot{x}_i = f_i(\kappa_i) x_i - f_i(\kappa_i) x_{i-1}^d + f_i(\kappa_i) e_{i-1}. \tag{6}$$

For $i \in \mathcal{T}$, substituting (6) into the time-derivative of (3) and using (3) yields

$$\dot{e}_3 = -f_3(\kappa_3) x_3 + f_3(\kappa_3) x_2^d - f_3(\kappa_3) e_2. \tag{7}$$

For $i \in \mathcal{M}$, substituting (6) into the time-derivative of (3) yields

$$\dot{e}_i = \dot{x}_i^d - f_i(\kappa_i) x_i + f_i(\kappa_i) x_{i-1}^d - f_i(\kappa_i) e_{i-1}. \tag{8}$$

For $i \in \mathcal{H}$, substituting (2) into the time-derivative of (3) yields

$$\dot{e}_0 = \dot{x}_0^d - d(x_0, t) - u. \tag{9}$$

The virtual trajectory for the mediator agent $i = 2$ is designed as

$$x_2^d \triangleq x_3 - k_3 e_3, \tag{10}$$

[2]The term v_r can be designed using numerous robust or adaptive methods.

[3]Due to space limitations, 3 agents are considered. All results contained herein extend without loss of generality to N agents.

where $k_3 \in \mathbb{R}_{>0}$ is a user-defined gain. The virtual trajectory in (10) is designed such that the mediator agent $i = 2$, is positioned to influence the target agent towards the goal location. Substituting (10) into (7) and simplifying yields

$$\dot{e}_3 = -f_3(\kappa_3)k_3e_3 - f_3(\kappa_3)e_2. \tag{11}$$

For the next mediator agent $i = 1$ in the chain to influence the mediator agent $i = 2$, a virtual trajectory is designed as

$$x_1^d \triangleq x_2 - K_2e_2, \tag{12}$$

where $K_2 \triangleq \frac{k_2}{\underline{f_2}}(\nu_1 + 1)$, $k_2 \in \mathbb{R}_{>0}$ is a user-defined gain, $\underline{f_2}$ is defined as in Assumption 1, and $\nu_1 \triangleq \overline{f_3}(k_3^2 + k_3)$. Substituting (12) into (8) for the mediator agent $i = 2$, yields

$$\dot{e}_2 = \dot{x}_2^d - K_2f_2(\kappa_2)e_2 - f_2(\kappa_2)e_1. \tag{13}$$

Substituting the time-derivative of (10) into (13) yields

$$\begin{aligned}\dot{e}_2 = {} & k_3f_3(\kappa_3)(k_3 + 1)e_3 \\ & + (f_3(\kappa_3)(k_3 + 1) - K_2f_2(\kappa_2))e_2 - f_2(\kappa_2)e_1.\end{aligned} \tag{14}$$

For the herding agent to track a trajectory to influence the mediator agent $i = 1$, a virtual trajectory is designed as

$$x_0^d \triangleq x_1 - K_1e_1, \tag{15}$$

where $K_1 \triangleq \frac{k_1}{\underline{f_1}}(\nu_2 + \nu_3 + 1)$, $k_1 \in \mathbb{R}_{>0}$ is a user-defined gain, $\nu_2 \triangleq K_2\overline{f_3}(k_3^2 + k_3)$, and $\nu_3 \triangleq K_2(\overline{f_2}(K_2 + 1) + \overline{f_3}(k_3 + 1))$. Substituting (15) into (8) for $i = 1$ yields

$$\dot{e}_1 = \dot{x}_1^d - K_1f_1(\kappa_1)e_1 - f_1(\kappa_1)e_0. \tag{16}$$

Substituting the time-derivative of (12) into (16) and simplifying yields

$$\begin{aligned}\dot{e}_1 = {} & K_2k_3f_3(\kappa_3)(k_3 + 1)e_3 \\ & + K_2(f_2(\kappa_2)(K_2 + 1) - f_3(\kappa_3)(k_3 + 1))e_2 \\ & + (f_2(\kappa_2)(K_2 + 1) - K_1f_1(\kappa_1))e_1 - f_1(\kappa_1)e_0.\end{aligned} \tag{17}$$

Substituting the time derivative of (15) into (9) yields

$$\begin{aligned}\dot{e}_0 = {} & K_1K_2k_3f_3(\kappa_3)(k_3 + 1)e_3 \\ & + K_1K_2(f_3(\kappa_3)(k_3 + 1) - f_2(\kappa_2)(K_2 + 1))e_2 \\ & + (K_1(f_1(\kappa_1)(K_1 + 1) - f_2(\kappa_2)(K_2 + 1)))e_1 \\ & + f_1(\kappa_1)(K_1 + 1)e_0 - d(x_0, t) - u.\end{aligned} \tag{18}$$

To yield the chained influence so that the target agent converges to the goal location, the herder controller with the structure seen in (5) is designed as

$$u = \beta_3e_3 + \beta_2e_2 + \beta_1e_1 + K_0e_0 + v_r, \tag{19}$$

where $\beta_3 \triangleq K_1K_2k_3(k_3 + 1)$, $\beta_2 \triangleq K_1K_2(k_3 - K_2)$, $\beta_1 \triangleq K_1(K_1 - K_2)$, $K_0 \triangleq k_0(\nu_4 + \nu_5 + 1)$, $k_0 \in \mathbb{R}_{>0}$ is a user-defined gain, $\nu_4 \triangleq (\overline{f_3} + \overline{f_2})(K_1K_2 + 1)$, and $\nu_5 \triangleq \overline{f_1}K_1 + \overline{f_2}(K_1 + 1)$. Additionally, as indicated in Footnote 2, v_r can be defined in various ways to compensate for different types of disturbances and uncertainties. Here, we use sliding mode control where $v_r \triangleq k_r\text{sgn}(e_0)$ where $k_r \geq \|\overline{d}\|$. Substituting (19) into (18) yields

$$\begin{aligned}\dot{e}_0 = {} & f_3(\kappa_3)e_3 + ((f_3(\kappa_3) - f_2(\kappa_2))(K_1K_2 + 1))e_2 \\ & + ((f_1(\kappa_1) - f_2(\kappa_2))(K_1 + 1))e_1 \\ & + (f_1(\kappa_1)(K_1 + 1) - K_0)e_0 - d(x_0, t) - v_r.\end{aligned} \tag{20}$$

Remark 1: In practical applications, agents may be subjected to random time-varying disturbances. For random disturbances with bounded magnitude, high-gain control terms could be used. To fully characterize the impact of such disturbances, a stochastic analysis can be performed yielding a convergence result in probability [24].

IV. Stability Analysis

Define the concatenated state vector $\zeta \in \mathbb{R}^{4n}$ where $\zeta \triangleq [e_3^\top, e_2^\top, e_1^\top, e_0^\top]^\top$. Using (11), (14), (17), and (20) yields the closed-loop error system

$$\dot{\zeta} = \begin{bmatrix} (-f_3(\kappa_3)(k_3e_3 + e_2)) \\ \begin{pmatrix} k_3f_3(\kappa_3)(k_3 + 1)e_3 \\ +(f_3(\kappa_3)(k_3 + 1) - K_2f_2(\kappa_2))e_2 \\ -f_2(\kappa_2)e_1 \end{pmatrix} \\ \begin{pmatrix} K_2k_3f_3(\kappa_3)(k_3 + 1)e_3 \\ +K_2(f_2(\kappa_2)(K_2 + 1))e_2 \\ -K_2f_3(\kappa_3)(k_3 + 1)e_2 \\ +(f_2(\kappa_2)(K_2 + 1) - K_1f_1(\kappa_1))e_1 \\ -f_1(\kappa_1)e_0 \end{pmatrix} \\ \begin{pmatrix} f_3(\kappa_3)e_3 + (f_1(\kappa_1)(K_1 + 1) - K_0)e_0 \\ -d(x_0, t) - v_r \\ +((f_3(\kappa_3) - f_2(\kappa_2))(K_1K_2 + 1))e_2 \\ +((f_1(\kappa_1) - f_2(\kappa_2))(K_1 + 1))e_1 \end{pmatrix} \end{bmatrix}. \tag{21}$$

To facilitate the stability analysis, consider the positive definite Lyapunov candidate $V : \mathbb{R}^{4n} \to \mathbb{R}_{\geq 0}$ defined as

$$V(\zeta) \triangleq \frac{1}{2}\zeta^\top\zeta. \tag{22}$$

Based on the definitions of ζ, the sufficient conditions are defined as

$$\lambda_{\min}\{Z\} > 0, \tag{23}$$

where $Z \triangleq \text{diag}\{k_3 - \frac{1}{\underline{f_3}}(\frac{\nu_1}{4k_2} + \frac{\nu_2}{4k_1} + 1), k_2 - (\overline{f_3}(k_3 + 1) + \frac{\overline{f_3}^2 + 1}{2} + \frac{\nu_3}{4k_1} + \frac{\nu_4}{4k_0}), k_1 - (\overline{f_2}(K_2 + 1) + \frac{\overline{f_2}^2}{2} + \frac{\nu_5}{4k_0}), k_0 - (\overline{f_1}(K_1 + 1) + \frac{\overline{f_3}^2}{2})\}$, a constant $\rho \in \mathbb{R}_{>0}$ is defined as $\rho \triangleq \text{diag}\{\frac{1}{\underline{f_3}}(\frac{\nu_1}{4k_2} + \frac{\nu_2}{4k_1} + 1), \overline{f_3}(k_3 + 1) + \frac{\overline{f_3}^2 + 1}{2} + \frac{\nu_3}{4k_1} + \frac{\nu_4}{4k_0}, \overline{f_2}(K_2 + 1) + \frac{\overline{f_2}^2}{2} + \frac{\nu_5}{4k_0}, \overline{f_1}(K_1 + 1) + \frac{\overline{f_3}^2}{2}\}$, and a rate of convergence $\lambda_c \in \mathbb{R}_{>0}$ is defined as

$$\lambda_c \triangleq \lambda_{\min}\{Z\} - \rho. \tag{24}$$

Theorem 1: Consider the dynamical systems described by (1) and (2). The controller given by (19) ensures that ζ is globally exponentially stable as

$$\|\zeta(t)\| \leq \|\zeta(t_0)\|\mathrm{e}^{-\lambda_c}(t - t_0) \tag{25}$$

$\forall t \in [t_0, \infty)$, provided that the control gains k_0, k_1, k_2, and k_3 are selected such that $\lambda_{\min}\{Z\} > 0$.

Proof: Substituting (21) into the time-derivative of (22) yields

$$\begin{aligned}\dot{V} = {} & -k_3f_3(\kappa_3)e_3^\top e_3 - K_2f_2(\kappa_2)e_2^\top e_2 - K_1f_1(\kappa_1)e_1^\top e_1 \\ & - K_0e_0^\top e_0 + f_3(\kappa_3)(k_3(k_3 + 1) - 1)e_3^\top e_2 \\ & - K_2k_3f_3(\kappa_3)(k_3 + 1)e_1^\top e_3 + f_3(\kappa_3)(k_3 + 1)e_2^\top e_2 \\ & + K_2(f_2(\kappa_2)(K_2 + 1) - f_3(\kappa_3)(k_3 + 1))e_1^\top e_2 \\ & - f_2(\kappa_2)e_1^\top e_2 + (f_3(\kappa_3) - f_2(\kappa_2))(K_1K_2 + 1)e_0^\top e_2 \\ & + f_3(\kappa_3)e_0^\top e_3 + (K_1f_1(\kappa_1) - f_2(\kappa_2)(K_1 + 1))e_0^\top e_1 \\ & + f_1(\kappa_1)(K_1 + 1)e_0^\top e_0 + f_2(\kappa_2)(K_2 + 1)e_1^\top e_1 \\ & - e_0^\top d(x_0, t) - e_0^\top v_r.\end{aligned} \tag{26}$$

Bounding the influencing functions $f_i(\kappa_i)$ for $i \in \mathcal{I}$, the herding dynamics $d(x_0, t)$, and consolidating the coefficients using $\nu_1, \nu_2, \nu_3, \nu_4$, and ν_5 yields

$$\begin{aligned}\dot{V} \leq& -k_3\underline{f}_3 e_3^\top e_3 - K_2\underline{f}_2 e_2^\top e_2 - K_1\underline{f}_1 e_1^\top e_1 - K_0 e_0^\top e_0 \\ &+ \overline{f}_1(K_1+1)e_0^\top e_0 + \overline{f}_2(K_2+1)e_1^\top e_1 + \nu_1 e_3^\top e_2 \\ &+ \overline{f}_3(k_3+1)e_2^\top e_2 + \nu_2 e_1^\top e_3 + \nu_3 e_1^\top e_2 + \nu_4 e_0^\top e_2 \\ &+ \nu_5 e_0^\top e_1 + \overline{f}_3 e_3^\top e_2 + \overline{f}_2 e_1^\top e_2 + \overline{f}_3 e_0^\top e_3 + e_0^\top \overline{d} - e_0^\top v_r. \end{aligned} \quad (27)$$

Expanding the definition of the gains K_0, K_1, and K_2 and applying the definition of v_r in (27) yields

$$\begin{aligned}\dot{V} \leq& \left(\overline{f}_1(K_1+1) - k_0(\nu_4+\nu_5+1)\right)e_0^\top e_0 \\ &+ \left(\overline{f}_2(K_2+1) - k_1(\nu_2+\nu_3+1)\right)e_1^\top e_1 \\ &+ \left(\overline{f}_3(k_3+1) - k_2(\nu_1+1)\right)e_2^\top e_2 - k_3\underline{f}_3 e_3^\top e_3 \\ &+ \nu_1 e_2^\top e_3 + \nu_2 e_1^\top e_3 + \nu_3 e_1^\top e_2 + \nu_4 e_0^\top e_2 \\ &+ \nu_5 e_0^\top e_1 + \overline{f}_3 e_3^\top e_2 + \overline{f}_2 e_1^\top e_2 + \overline{f}_3 e_0^\top e_3 \\ &+ e_0^\top \overline{d} - k_r e_0^\top \operatorname{sgn}(e_0). \end{aligned} \quad (28)$$

Applying Young's inequality to $\overline{f}_3 e_3^\top e_2 + \overline{f}_2 e_1^\top e_2 + \overline{f}_3 e_0^\top e_3$ yields

$$\begin{aligned} \overline{f}_3 e_3^\top e_2 &\leq \frac{\|e_3\|^2}{2} + \frac{\overline{f}_3^2\|e_2\|^2}{2}, \\ \overline{f}_2 e_1^\top e_2 &\leq \frac{\|e_2\|^2}{2} + \frac{\overline{f}_2^2\|e_1\|^2}{2}, \\ \overline{f}_3 e_0^\top e_3 &\leq \frac{\|e_3\|^2}{2} + \frac{\overline{f}_3^2\|e_0\|^2}{2}. \end{aligned} \quad (29)$$

Completing the squares on the remaining sign-indefinite terms in (28) yields

$$\begin{aligned} \nu_1\|e_3\|\|e_2\| - \nu_1 k_2\|e_2\|^2 &\leq \frac{\nu_1}{4k_2}\|e_3\|^2, \\ \nu_2\|e_3\|\|e_1\| - \nu_2 k_1\|e_1\|^2 &\leq \frac{\nu_2}{4k_1}\|e_3\|^2, \\ \nu_3\|e_2\|\|e_1\| - \nu_3 k_1\|e_1\|^2 &\leq \frac{\nu_3}{4k_1}\|e_2\|^2, \\ \nu_4\|e_0\|\|e_2\| - \nu_4 k_0\|e_0\|^2 &\leq \frac{\nu_4}{4k_0}\|e_2\|^2, \\ \nu_5\|e_0\|\|e_1\| - \nu_5 k_0\|e_0\|^2 &\leq \frac{\nu_5}{4k_0}\|e_1\|^2. \end{aligned} \quad (30)$$

Using (29) and (30) and the definition of k_r, we can upper bound (28) as

$$\begin{aligned}\dot{V} \leq& -\left(k_0 - \overline{f}_1(K_1+1) - \frac{\overline{f}_3^2}{2}\right)\|e_0\|^2 \\ &- \left(k_1 - \overline{f}_2(K_2+1) - \frac{\overline{f}_2^2}{2} - \frac{\nu_5}{4k_0}\right)\|e_1\|^2 \\ &- \left(k_2 - \overline{f}_3(k_3+1) - \frac{\overline{f}_3^2+1}{2} - \frac{\nu_3}{4k_1} - \frac{\nu_4}{4k_0}\right)\|e_2\|^2 \\ &- \left(k_3\underline{f}_3 - \frac{\nu_1}{4k_2} - \frac{\nu_2}{4k_1} - 1\right)\|e_3\|^2. \end{aligned} \quad (31)$$

Applying the sufficient conditions (23), and rewriting (31) with (22) and ρ yields

$$\dot{V} \leq -(\lambda_{\min}\{Z\} - \rho)\|\zeta\|^2. \quad (32)$$

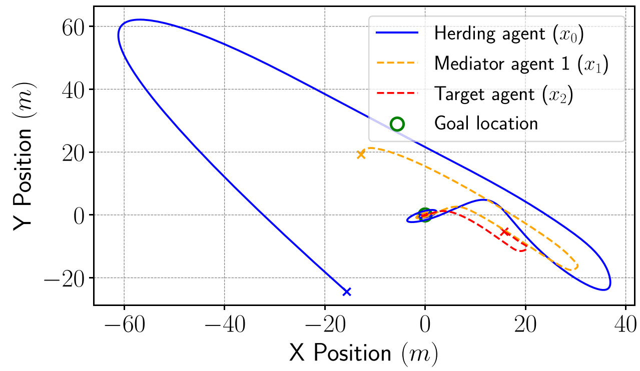

Fig. 2. Trajectories of the herding agent (blue), the mediator agent (orange), and the target agent (red). The ×'s represent the initial positions of each agent. The goal location is set at the origin.

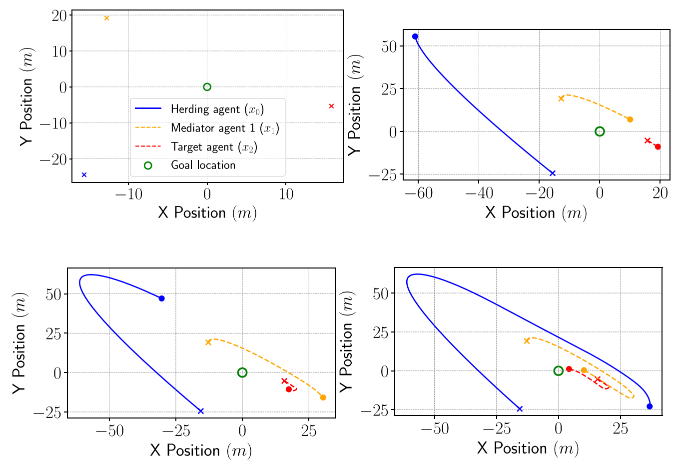

Fig. 3. Trajectories of the herding agent, mediator agent, and the target agent as a series of snapshots in time starting from the upper left and progressing left to right and top to bottom to the bottom right figure. The initial positions are depicted by ×'s and current positions of the respective agents are depicted by the ∘'s.

Applying the definition (24) to (32) yields the negative definite inequality

$$\dot{V}(\zeta) \leq -2\lambda_c V(\zeta). \quad (33)$$

Solving the differential inequality in (33) yields

$$V(\zeta(t)) \leq V(\zeta(t_0))\mathrm{e}^{-2\lambda_c(t-t_0)}. \quad (34)$$

Substituting (22) into (34) yields (25) proving $\zeta \in \mathcal{L}_\infty$. Since $\zeta \in \mathcal{L}_\infty$, then $e_0, e_1, e_2, e_3 \in \mathcal{L}_\infty$. Thus, by using (19), $u \in \mathcal{L}_\infty$. ■

Remark 2: Extending the chain of agents to N will yield gain conditions that grow due to the cascading nature of the backstepping design. Specifically, the required gain for an agent compensates for the gain condition of the immediately preceding agent (i.e., the agent closer to the target agent). Consequently, agents further away from the target agent, need to be more agile to maneuver and influence the subsequent agent. Future designs are motivated to mitigate the correlation between gain escalation and the total number of agents in the chain.

V. Simulations

To better visualize the herding strategy, an example with $N = 2$ agents is shown in Figures 2 and 3. The herding

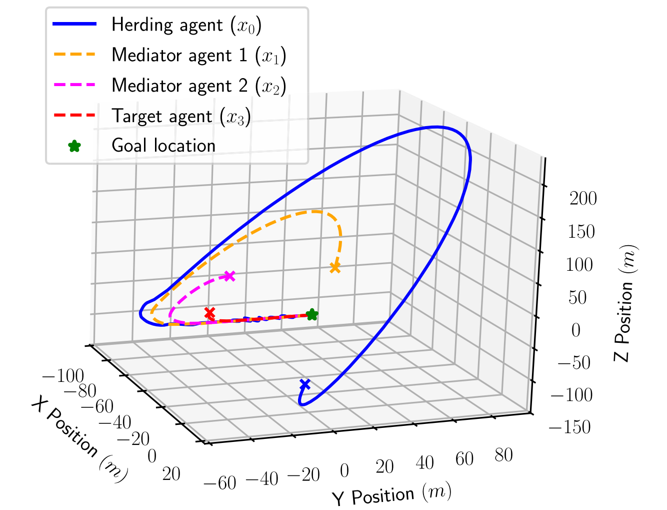

Fig. 4. Trajectories of the herding agent, mediator agents, and target agent after 14.5 seconds. The goal location is set at the origin. The initial positions of each agent are randomly selected from a uniform distribution $U(-50, 50)$ and represented by an "×".

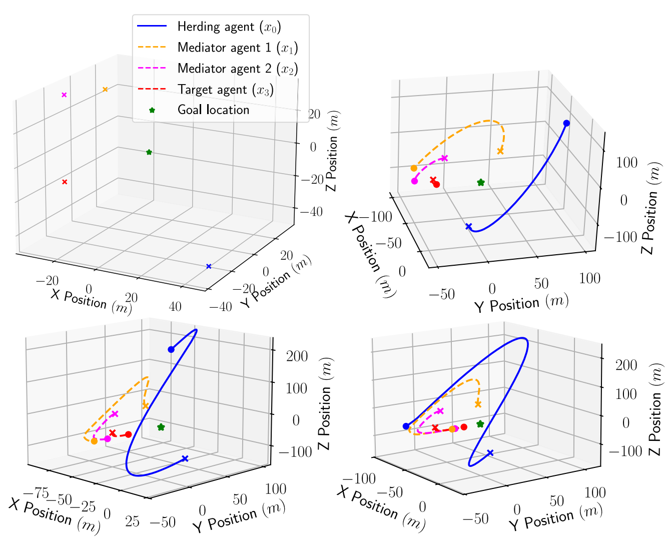

Fig. 5. Trajectories of the herding agent, two mediator agents, and the target agent as a series of snapshots in time starting from the upper left and progressing left to right and top to bottom to the bottom right figure. The initial positions are depicted by ×'s and current positions of the respective agents are depicted by the ∘'s.

agent for this case does not have additional drift dynamics resulting in the gain k_r being omitted. The unknown interaction function, f_i, for the agents within the chain of influence were defined in simulation as $f_i(x_i, x_{i-1}) = \exp(-\frac{\|x_i - x_{i-1}\|^2}{\alpha}) + \Psi$, where $\alpha \in \mathbb{R}_{>0}$ is a constant affecting the radius of influence and Ψ is the lower bound of the influencing function of the i^{th} agent. The influencing functions for the agents in Figures 2 and 3 were set as $f_2(x_2, x_1) = \exp(-\frac{\|x_2 - x_1\|^2}{20}) + 0.25$ and $f_1(x_1, x_0) = \exp(-\frac{\|x_1 - x_0\|^2}{10}) + 0.45$. Snapshots of the simulation for this case at different points in time are displayed in Figure 3. The selected gains were $k_2 = 1.5$, $k_1 = 3.25$, and $k_0 = 6$.

To further illustrate the presented results, a simulation was performed for the case $N = 3$ with results presented in Figures 4-6. The influencing chain consists of a herding agent,

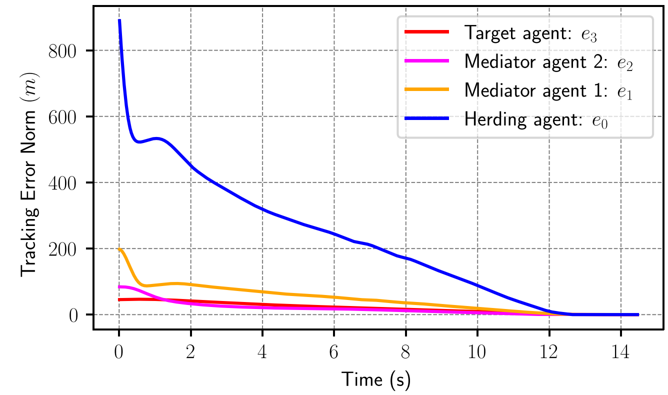

Fig. 6. The simulated tracking errors for each agent are shown to converge to zero. The tracking error of the herding agent (blue), mediator agent 1 (orange), mediator agent 2 (magenta), and the target agent (red) are displayed.

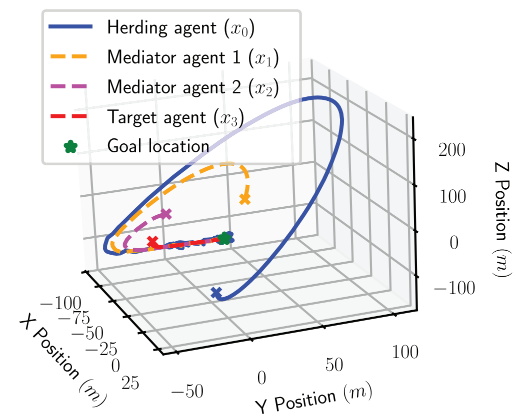

Fig. 7. Trajectories of the herding agent, mediator agents, and target agent after 30 seconds with initial positions of each agent represented by an "×".

two mediator agents, and a target agent. The tracking errors for the herding agent, first mediator agent, second mediator agent, and target agent are denoted by e_0, e_1, e_2, and e_3, respectively. The influencing functions for the agents in the simulation were set as $f_3(x_3, x_2) = \exp(-\frac{\|x_3 - x_2\|^2}{20}) + 0.25$, $f_2(x_2, x_1) = \exp(-\frac{\|x_2 - x_1\|^2}{40}) + 0.5$, and $f_1(x_1, x_0) = \exp(-\frac{\|x_1 - x_0\|^2}{30}) + 1$. The herding agent dynamics include a bounded disturbance defined as $d(x_0, t) = [1.5\cos(x) - 2\sin(y), 1.5\cos(z), 2\sin(y), \cos(z) + \sin(x)]^\top$. A progression of the simulation is depicted in Figure 4, where the position of each agent at $t = 0$ sec, $t = 1.5$ sec, $t = 3.5$ sec, and $t = 7.5$ sec are shown in Figure 5. The selected gains for this simulation case were $k_3 = 1.3$, $k_2 = 2.7$, $k_1 = 4.8$, $k_0 = 7$, and $k_r = 7$. The desired trajectory of agents $i \in \mathcal{I}$ cascade through the closed-loop errors for every succeeding agent in the chain. In Figure 6, the cascading nature of the desired trajectories can be seen in the agent's tracking errors. The tracking error e_0 has the largest initial error throughout the simulation, and the subsequent errors e_i in the chain decrease as i approaches $N = 3$.

For a noisy representation of the mediator agents and target agent dynamics an additional simulation was performed with results presented in 7 and 8. Specifically, the dynamics for the agents $i \in \mathcal{I}$ were disturbed by random noise sampled from a normal distribution $\mathcal{N}(0, 1)$. The initial conditions, goal location, and gain parameters matched those of the noise-free

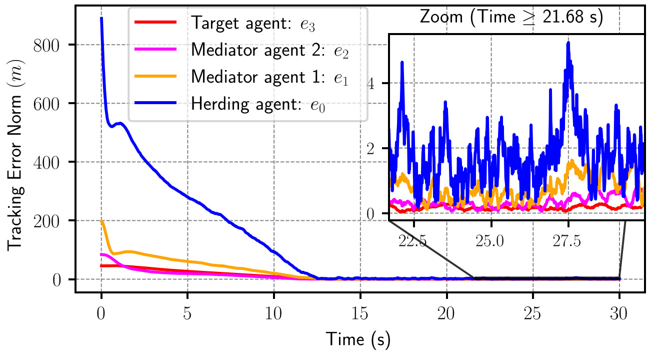

Fig. 8. The tracking error of the herding agent (blue), mediator agent 1 (orange), mediator agent 2 (magenta), and the target agent (red) are shown over the 30 second simulation. A magnified plot highlights the steady-state effects of the added white Gaussian noise.

$N = 3$ case, while the herding agent dynamics and influencing functions remained unchanged.

VI. Conclusion

This letter addresses an indirect herding control problem with a chain of influencing agents, where a herding agent is tasked with indirectly influencing a series of mediator agents to regulate a target agent towards a goal location. Due to the structure of the multi-agent interaction chain, which prevents direct control of all agents, robust control methods are used to compensate for uncertainty in the influence function that dictates how each agent influences subsequent agents, including the target agent to a desired goal location. A Lyapunov-based stability analysis is used to prove the developed controller yields exponential regulation of the target agent to its goal location. Simulations demonstrate the efficacy of the result, even with the inclusion of added noise. Future work could explore the integration of learning-based methods to compensate for the uncertainty in the chain of influence. Additional efforts could also explore other topological considerations for exerting influence where multiple agents within a graphical architecture can provide distributed and partial influence.

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