Optimization-Based Controllers for Passivity and Safety Constraints

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***Abstract*—Though historically regarded as unrelated concepts, passivity-based control (PBC) and control barrier functions (CBFs) are methods used to establish the safety of control systems. In this article, an optimization-based PBC technique is developed which can be combined with CBF-based methods to find a set of controllers that each render a nonlinear control system energetically passive while also adhering to state constraints necessary for safety. Borrowing concepts from literature developed for multiple CBFs, passivity and safety objectives are simultaneously achieved through the use of a pointwise-optimal controller. Simulation results demonstrate the closed-loop system is passive with respect to an external disturbance despite a nonpassive nominal control input while also satisfying state constraints required for safety.**

***Index Terms*—Control barrier functions (CBFs), optimization, passivity-based control (PBC), quadratic programming.**

I. Introduction

Passivity-based control (PBC) and control barrier functions (CBFs) are two common methods of ensuring the safety of nonlinear control systems [1], [2], [3], though the definition of safety differs in each context. In passivity (and more generally dissipativity) theory, systems are viewed from an energy perspective, where complex systems can be broken down into simpler components and individually evaluated to produce a system that is stable despite unknown dynamics or interconnection with an unknown environment [4]. Separately, CBFs are a tool used to constrain the system to some allowable subset of the state space, preventing the system from reaching states that are deemed unsafe [5]. Though PBC and CBFs have typically been regarded as independent concepts, both PBC and CBFs have implications in safe control and can be combined to stabilize systems interacting with an unknown external disturbance while satisfying prescribed state constraints.

When defining safety in relation to PBC and CBFs, some nuance is required. Broadly, safety simply means that a bad event does not occur [6], [7], but there is flexibility in defining what that bad event is. In the context of PBC, there is no rigid mathematical definition of what is considered safe. Instead, in PBC, safety generally refers to ensuring robust stability with respect to the system's environment [8]. Contrarily, in CBF works, safety refers to a more specific mathematical notion, where a system is considered to be safe if it is restricted to a forward invariant set of safe states [3]. While PBC and CBF-based methods are typically unable to prevent all adverse events from occurring, each approach introduces different notions of a safe control action, each of which can be advantageous to enforce.

By definition, passive systems can only store or release the energy supplied to them [9]. Originating in circuit theory [10], [11], PBC of rigid manipulators was introduced in [12] and has since been applied to a variety of mechanical systems, including multiagent systems [13], [14], underactuated systems [15], [16], bipedal locomotion [17], and bilateral teleoperators [18], as well as other systems requiring human–machine interaction [19], [20]. Nonpassive control action can disrupt the passivity of an otherwise passive closed-loop system. Passivity is a desirable property since controllers that render a system passive can provide robust stability guarantees to systems interfacing with an unknown environment, subject to human input, or experiencing time delay [1]. Typically, passivity-ensuring controllers are designed constructively, similar to Lyapunov-based approaches, where one specific controller is found that satisfies the passivity condition.

Unlike in traditional PBC, CBF-based approaches do not yield a singular controller that will produce the desired behavior, but rather a set of controllers that all satisfy some constraint. CBFs are typically used to enforce the forward invariance of some safe subset of the state space, meaning that trajectories that start inside the safe set stay there for all time [5]. In CBF literature, selections from the safe set of control inputs are commonly made through the use of a quadratic program (QP) [21]. Optimization methods like QPs are used to make a selection from the allowable set of control inputs that minimizes some cost function.

When combining passivity and state constraints, literature developed for multiple CBFs in [22], [23], and [24] can be a powerful tool. By reformulating the passivity constraint into a CBF-like constraint on the control input, optimization techniques can be used to synthesize a set of controllers that yield overall passivity of the system. Construction of the specific controller that yields passivity or forward invariance of the safe set is not required. Instead, a QP can be used to select a control input from a set of safe passivating controllers for each point in the state space, enabling easier integration with potential CBF-based state constraints.

Several works have explored combining the concepts of PBC and CBFs [25], [26], [27], [28], [29]. The authors in both [25] and [26] used optimization-based methods to passivate nonpassive control actions, and the authors in [28] and [29] introduced control storage functions and control dissipation functions as aids to design stabilizing controllers for receding horizon control problems. In [25], the energy tank framework introduced in [8] was used to model the flow of energy in the system. In each of these works, optimization techniques are used only to enforce passivity [25], [26] or dissipativity [28], [29] but do not consider any state-based safety constraints. Califano [27] developed conditions under which a passive controller remains passive after being modified by a safety-filtering QP. Despite using optimization techniques to enforce safety constraints, the result in [27] requires the initial design of a specific nominal controller that renders the system passive. Furthermore, Califano [27] neglected the effect of the external disturbance on the evolution of the state in the design of the CBF constraint. As a result, there is a potential for these external disturbances to compromise the forward invariance of the safe set.

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This article combines the ideas of PBC and multiple CBFs to design an optimization-based controller that renders the closed-loop system passive and a safe set forward invariant despite an external disturbance. By using a QP to enforce both passivity and safety constraints, a set of allowable controllers is developed, generalizing the control design while providing performance guarantees. While the developed passivity constraint resembles a Lyapunov constraint, PBC and Lyapunov-based control are separate concepts with separate applications. Previous results combining PBC and CBFs require the initial design of a passive nominal controller and provide conditions for which the passivity of that specific nominal controller is not disrupted by a safety constraint, while the developed technique produces a set of passivating and safety-ensuring controllers. The developed approach results in a forward invariant safe set that is robust to the external disturbance. In addition, we provide a method to determine the feasibility of the synthesized controller using sum of squares programming. Simulation results on a planar two-link robot disturbed by an interaction torque injected by a human operator demonstrate the ability of the developed approach to achieve both passivity and safety objectives.

A. Notation and Preliminaries

Given vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, $(x, y) \triangleq [x^\top, y^\top]^\top$. The shorthand $[d] \triangleq \{1, 2, \ldots, d\}$ denotes the first d positive integers. For a function $B : \mathbb{R}^n \to \mathbb{R}^d$, the components are indexed as $B(x) \triangleq [B_1(x), B_2(x), \ldots, B_d(x)]^\top$, and the inequality $B(x) \leq 0$ means that $B_i(x) \leq 0$ for all $i \in [d]$. For a set $\mathcal{S} \subset \mathbb{R}^n$, the notation $\partial\mathcal{S}$ represents its boundary and $\mathcal{N}(\mathcal{S})$ represents an open neighborhood about $\mathcal{S}$.

For a set $X \subset \mathbb{R}^n$, the set-valued mapping $G : X \rightrightarrows \mathbb{R}^m$ associates every point $x \in X$ with a set $G(x) \subset \mathbb{R}^m$. The mapping G is called locally bounded if, for every $x \in X$, there exists a neighborhood $\mathcal{N}_X(x) \triangleq \mathcal{N}(x) \cap X$ such that $G(\mathcal{N}_X(x))$ is bounded, and G is outer semicontinuous if $\mathrm{Graph}_X(G) \triangleq \{(x, u) \in X \times \mathbb{R}^m : u \in G(x)\}$ is relatively closed in $X \times \mathbb{R}^m$.

II. System Model

Consider a control system in the form

$$\dot{x} = f(x, \nu) + g(x)u \tag{1}$$

where the state is denoted $x \in \mathbb{R}^n$, the control input is denoted by $u \in \Psi(x) \subset \mathbb{R}^m$, an external disturbance is denoted by $\nu \in \Phi(x) \subset \mathbb{R}^p$, and known continuous functions are denoted by $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$. The set-valued mappings $\Psi : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ and $\Phi : \mathbb{R}^n \rightrightarrows \mathbb{R}^p$ represent the admissible values for the input and external disturbance, respectively, at each state. The control input u has state-dependent constraints, so to develop an implementable controller we impose the following assumption.

Assumption 1: There exists a function $\psi : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^s$ such that $\Psi(x) = \{u \in \mathbb{R}^m : \psi(x, u) \leq 0\}$ is nonempty.

III. CBFs for State Constraints

The physical safety of a system is commonly defined by state constraints that must be satisfied. For example, a system may have position or velocity limits that must be met for obstacle avoidance. CBFs are a method of converting those state constraints into constraints on the control input. A control input found through a CBF-based analysis renders the safe set forward invariant, i.e., trajectories that begin within the safe set remain there for all time [3]. Therefore, if the state begins in a safe region, it will be unable to reach an unsafe region of the state space. Given a controller κ, let the closed-loop dynamics defined by (1) and κ be defined as $\dot{x} = f_{cl}(x) \triangleq f(x, \nu) + g(x)\kappa(x)$. A solution to the closed-loop dynamics $t \mapsto x(t)$ is complete if $\mathrm{dom}x$ is unbounded and maximal if there is no solution x' such that $x(t) = x'(t)$ for all $t \in \mathrm{dom}x$, where $\mathrm{dom}x$ is a proper subset of $\mathrm{dom}x'$. The following definition formalizes the notion of forward (pre-)invariance.

Definition 1 ([30, Definitions 2.5 and 2.6]): The set $\mathcal{S} \subset \mathbb{R}^n$ is forward preinvariant for the closed-loop dynamics $\dot{x} = f_{cl}(x)$ if, for each $x_0 \in \mathcal{S}$ and each maximal solution x starting from x_0, $x(t) \in \mathcal{S}$ for all $t \in \mathrm{dom}x$. The set $\mathcal{S}$ is forward invariant for the closed-loop dynamics if it is forward preinvariant and, for each $x_0 \in \mathcal{S}$, every maximal solution x starting from x_0 is complete.

To enforce the forward invariance of the safe set, we must consider the effects of the external disturbance in (1). While the authors in [27] investigated the combination of passivity and state constraints, consideration of the external disturbance is omitted in the CBF development, resulting in the potential for the state to be pushed into an unsafe region of the state space by the external disturbance. Compared to previous results that combine passivity and CBFs, the subsequent development systematically considers the impact of the unknown external disturbance on the evolution of the state to ensure that it does not disrupt the forward invariance of the safe set. To do so, the following conditions are imposed on the system dynamics [24].

Assumption 2: The set-valued mapping $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ defined as $F(x) \triangleq \{f(x, \nu) : \nu \in \Phi(x)\}$ is nonempty, convex-valued, and bounded for every $x \in \mathbb{R}^n$.

Assumption 3: The set $\Phi(x) \subset \mathbb{R}^p$ is closed for every $x \in \mathbb{R}^n$.

We adapt the definition of a CBF presented in [24, Definition 2] to fit the dynamic system in (1). This definition considers a notion of vector-valued CBFs, where the safe set can be defined by multiple scalar-valued functions, corresponding to multiple state constraints. A function $B : \mathbb{R}^n \to \mathbb{R}^d$ is a CBF candidate defining the safe set $\mathcal{S} \subset \mathbb{R}^n$ if $\mathcal{S} = \{x \in \mathbb{R}^n : B(x) \leq 0\}$, where $B(x) \triangleq [B_1(x),\ B_2(x),\ \ldots,\ B_d(x)]^\top$. If B is continuous, $\mathcal{S}$ is a closed set. The scalar-valued CBF candidates denoted by $B_i : \mathbb{R}^n \to \mathbb{R}$ each define sets $\mathcal{S}_i \triangleq \{x \in \mathbb{R}^n : B_i(x) \leq 0\}$ and $M_i \triangleq \{x \in \partial\mathcal{S} : B_i(x) = 0\}$, for each $i \in [d]$.

For a continuously differentiable CBF candidate B, we define a function $\Gamma : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^d$ such that for each $i \in [d]$, the ith component of Γ is defined as

$$\Gamma_i(x, u) \triangleq \sup_{\nu \in \Phi(x)} \{\nabla B_i(x)^\top (f(x, \nu) + g(x)u)\}. \tag{2}$$

This definition of Γ accounts for the worst-case value of the external disturbance and is similar to what is commonly used in robust CBF literature [31], [32], [33], [34]. The function Γ_i represents the worst-case growth of $B_i(x)$ for any direction in the set-valued mapping $F(x)$. We also introduce a function $\gamma : \mathbb{R}^n \to \mathbb{R}^d$ defined as $\gamma(x) \triangleq [\gamma_1(x),\ \gamma_2(x),\ \ldots,\ \gamma_d(x)]^\top$ which is a user-selected function used to constrain the rate of growth of Γ to guarantee the forward invariance of $\mathcal{S}$.

Definition 2 ([24, Definition 2]): A continuously differentiable vector-valued CBF candidate $B : \mathbb{R}^n \to \mathbb{R}^d$ defining the set $\mathcal{S} \subset \mathbb{R}^n$ is a CBF for (1) and $\mathcal{S}$ on a set $\mathcal{O}_{\mathcal{C}} \subset \mathbb{R}^n$ with respect to a function $\gamma : \mathbb{R}^n \to \mathbb{R}^d$ if 1) there exists a neighborhood of the boundary of $\mathcal{S}$ such that $\mathcal{N}(\partial\mathcal{S}) \subset \mathcal{O}_{\mathcal{C}}$; 2) for each $i \in [d]$, $\gamma_i(x) \geq 0$ for all $x \in \mathcal{N}(M_i) \backslash \mathcal{S}_i$; and 3) the set

$$K_c(x) \triangleq \{u \in \Psi(x) : \Gamma(x, u) \leq -\gamma(x)\} \tag{3}$$

is nonempty for all $x \in \mathcal{O}_{\mathcal{C}}$.

Based on theoretical conditions for forward invariance in [24], the set-valued mapping K_c in (3) defines a set of control inputs that ensure safety. More specifically, Isaly et al. [24] showed that, when B is a CBF and some additional conditions are satisfied, continuous controllers selected from the mapping K_c render the safe set $\mathcal{S}$ forward invariant.

Remark 1: The function γ is similar to the functions used in works, such as [21], typically in the form of $\gamma_i(x) = \alpha(B_i(x))$, where α is an extended class-$\mathcal{K}$ function. The restrictions on γ presented in Definition 2 only require that γ is nonnegative in a region just outside the boundary of the safe set, instead of everywhere outside of the safe set, which is typically required. While the use of an extended class-$\mathcal{K}$ function can help to achieve asymptotic stability of the safe set, extended class-$\mathcal{K}$ functions often impose stronger conditions on the growth of Γ than are necessary to achieve forward invariance of the safe set, especially when the safe set is not compact. Therefore, using the notion of γ introduced in [24, Sec. II], where γ does not have to be explicitly dependent on the CBF, allows for additional design flexibility when selecting γ.

IV. Passivity-Based Control Development

If a system is passive, the output energy of the system can be no greater than the energy that is put into the system [35]. Therefore, PBC is commonly used in systems that interact with a potentially unknown environment as a way to ensure stable interaction between the system and environment. Based on typical definitions of passivity, such as the one in [9, Definition 6.3], the system in (1) is said to be passive from disturbance $\nu \in \Phi(x)$ to output $h : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^p$ if there exists a positive semidefinite continuously differentiable storage function $V : \mathbb{R}^n \to \mathbb{R}$ such that

$$\dot{V}\left(x, u, \nu\right) \leq \nu^\top h\left(x, \nu\right). \tag{4}$$

In a way similar to a Lyapunov stability analysis, this definition requires the design of a single passivating control input u that makes the inequality in (4) true. We modify the definition of passivity in [9] to present the notion of control passivity, resulting in a set of controllers that each render the system passive. The developed control passivity definition is compatible with optimization-based controller synthesis methods commonly used with CBFs, allowing for simpler unification of passivity and safety constraints.

Definition 3: For an output $y = h(x, \nu)$, the system in (1) is considered to be *controllably passive* with respect to input ν and output h on a set $\mathcal{O}_P \subseteq \mathbb{R}^n$ if there exists a continuously differentiable positive semidefinite storage function $V : \mathbb{R}^n \to \mathbb{R}$ such that the set

$$K_p(x) \triangleq \left\{ u \in \Psi(x) : \sup_{\nu \in \Phi(x)} \left\{ \dot{V}\left(x, u, \nu\right) - \nu^\top h\left(x, \nu\right) \right\} \leq 0 \right\} \tag{5}$$

is nonempty for every $x \in \mathcal{O}_P$, where $\dot{V}(x, u, \nu) \triangleq \nabla V(x)^\top (f(x, \nu) + g(x)u)$.

Remark 2: The storage function can also be vector-valued, defined as $V \triangleq [V_1, V_2, \ldots, V_b]^\top : \mathbb{R}^n \to \mathbb{R}^b$, rendering the system in (1) passive with respect to multiple input–output pairs. In this case, each storage function defines a constraint on the control input such that

$$\sup_{\nu \in \Phi(x)} \left\{ \dot{V}_a\left(x, u, \nu\right) - \nu_a^\top h_a\left(x, \nu\right) \right\} \leq 0 \quad \forall a \in [b].$$

By designing the set of passivity-ensuring controllers in (5), controller synthesis methods commonly used with CBFs can be used to enforce passivity. Selections from K_p ensure the system is passive in the same way selections from K_c render a safe set forward invariant.

A. Implementation

A new representation of passivity based on Definition 3 is developed to better integrate PBC with CBF mechanisms. To design a controller u that renders the closed-loop system passive with respect to the external disturbance, we develop a function $P : \mathbb{R}^n \to \mathbb{R}$ defined as

$$P(x) \triangleq \sup_{\nu \in \Phi(x)} \left\{ \nabla V(x)^\top f\left(x, \nu\right) - \nu^\top h\left(x, \nu\right) \right\}. \tag{6}$$

Note that P is finite when the system is controllably passive with storage function V. The set of passivating control inputs in (5) can be rewritten as

$$K_p(x) \triangleq \left\{ u \in \Psi(x) : \nabla V(x)^\top g(x) u \leq -P(x) \right\}. \tag{7}$$

Instead of designing a specific controller that renders the system passive, a selection from (7) can be made using an optimization-based control law. The control law takes the form of

$$\begin{aligned} \kappa^*(x) \triangleq\ & \underset{u \in \mathbb{R}^m}{\arg\min}\ Q(x, u) \\ & \text{s.t.} \nabla V(x)^\top g(x) u \leq -P(x) \\ & \psi(x, u) \leq 0 \end{aligned} \tag{8}$$

yielding a controller that is a selection from K_p while minimizing some cost function $Q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and satisfying the input constraints on u, assuming the cost function and constraints are selected such that the control input is unique. Conditions on the cost function and constraints that yield a single-valued and continuous controller are presented in the next section.

Example 1: We will demonstrate a few subtleties of the above development with a simple example. Begin with the system

$$\begin{cases} \dot{x} = x + u + \nu \\ y = x \end{cases} \tag{9}$$

where the state and control input are denoted by $x, u \in \mathbb{R}$, and the external disturbance is $\nu \in \mathbb{R}$. Selections of different storage functions can produce different values of P. By selecting the storage function as $V = \frac{1}{2}x^2$, the function P is defined as $P(x) \triangleq \sup_{\nu \in \Phi(x)} \{x^2 + x\nu - \nu x\} = x^2$ which is known and the system is therefore controllably passive with respect to input–output pair (ν, x) using Definition 3, regardless of the boundedness of $\Phi(x)$. For an unbounded external disturbance term, the above storage function is the only possible selection that results in a finite P. If the storage function was instead chosen as $V = x^2$, the function P is $P(x) \triangleq \sup_{\nu \in \Phi(x)} \{2x^2 + 2x\nu - \nu x\} = \sup_{\nu \in \Phi(x)} \{2x^2 + x\nu\}$ which may be unbounded for an unbounded $\Phi(x)$.

V. Passivity Preservation

The notion of control passivity is compatible with CBF literature and can be enforced using the same methods, meaning that passivity and safety constraints can each be used as a condition in one pointwise optimal controller. The set of passivity- and safety-ensuring control laws can be found at the intersection of K_p in (7) and K_c in (3) and is defined as $K(x) \triangleq K_p(x) \cap K_c(x)$. A selection from K that minimizes some cost function can be implemented using the controller

$$\begin{aligned} \kappa^*(x) \triangleq\ & \underset{u \in \mathbb{R}^m}{\arg\min}\ Q(x, u) \\ & \text{s.t. } \Gamma(x, u) \leq -\gamma(x) \\ & \nabla V(x)^\top g(x) u \leq -P(x) \\ & \psi(x, u) \leq 0 \end{aligned} \tag{10}$$

where the cost function $Q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is often chosen as $Q = \left\| u - u_{\text{nom}}(x) \right\|^2$ to minimally modify some continuous nominal control input $u_{\text{nom}} : \mathbb{R}^n \to \mathbb{R}^m$, again assuming that the cost function and constraints are selected such that the control input is unique.

Definitions 2 and 3 provide conditions for when the control system is rendered passive or a set is rendered forward invariant, respectively. The optimization problem in (10) yields a controller that is both passive and safe without the a priori design of a passive nominal controller. For the system to be both passive and safe, there must be at least one control input for each $x \in \mathbb{R}^n$ that satisfies both conditions. Thus, passivity can be preserved in the presence of safety constraints using the QP in (10) only when the constraints are simultaneously feasible.

Although $\kappa^*(x)$ is feasible if $K(x) \neq \emptyset$, the optimization problem in (10) does not necessarily generate a single-valued and continuous controller. The following lemma of [24, Lemma 3] provides conditions on the cost function and constraints that will result in a single-valued and continuous control input. The lemma is presented in a generic form for simplicity.

Lemma 1 ([24, Lemma 3]): Let $\mathcal{C} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^k$ be continuous on $\mathcal{O} \times \mathbb{R}^m$, and, for each $j \in [k]$, let $u \mapsto \mathcal{C}_j(x, u)$ be convex on the set $K(x) \triangleq \{u \in \mathbb{R}^m : \mathcal{C}_j(x, u) \leq 0 \quad \forall j \in [k]\}$. Suppose $Q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is continuous and, for each $x \in \mathcal{O}$, $u \mapsto Q(x, u)$ is strictly convex and inf-compact[1] on $K(x)$. If the set $K^\circ(x) \triangleq \{u \in \mathbb{R}^m : \mathcal{C}_j(x, u) < 0 \quad \forall j \in [k]\}$ is nonempty for every $x \in \mathcal{O}$, $\kappa^*(x) \triangleq \arg\min_{u \in K} Q(x, u)$ is single-valued and continuous.

We impose the following assumption on the constraints in (10) to yield the necessary continuity properties required by Lemma 1. The assumption allows the main result establishing the forward (pre-) invariance of the safe set $\mathcal{S}$ and passivity of the closed-loop system from the external disturbance ν to output y to be proven.

Assumption 4: For all $i \in [d]$ and $r \in [s]$, the functions $u \mapsto \Gamma_i(x, u)$ in (2) and $u \mapsto \psi_r(x, u)$ defining the set Ψ in Assumption 1 are convex on the set $K_c(x)$ for all $x \in \mathcal{O}_C$. For all $i \in [d]$ and $r \in [s]$, the functions $(x, u) \mapsto \Gamma_i(x, u) + \gamma_i(x)$ defining the set of safety ensuring control inputs and $(x, u) \mapsto \psi_r(x, u)$ are each continuous on $\mathcal{O}_C \times \mathbb{R}^m$. The function $(x, u) \mapsto \nabla V(x)^\top g(x) u + P(x)$ is continuous on $\mathcal{O}_P \times \Psi$.

Recall that the notion of forward preinvariance allows for maximal solutions to the closed-loop system that are not complete. The following assumption helps to establish the stronger notion of forward invariance of $\mathcal{S}$.

Assumption 5: Maximal solutions to the closed-loop system defined by (1) and (10) cannot escape in finite-time inside the safe set $\mathcal{S}$.

Theorem 1: Consider the system in (1) with a control input selected by the passivity- and safety-ensuring optimization problem in (10). Suppose Assumptions 1–4 hold, the function B is a CBF defining the set $\mathcal{S}$, the function V is a positive semidefinite storage function, and $\mathcal{O}_P \supset \mathcal{N}(\mathcal{S})$. Let the cost function Q be continuous, $u \mapsto Q(x, u)$ be strictly convex for each $x \in \mathcal{O}_P$, and, for each $x \in \mathcal{O}_P$, let $u \mapsto Q(x, u)$ be strictly convex and inf-compact on $K(x)$. In addition, let the mapping

$$K^\circ(x) \triangleq \left\{ \begin{matrix} u \in \mathbb{R}^m : \Gamma(x, u) < -\gamma(x) \\ \nabla V(x)^\top g(x) u < -P(x) \\ \psi(x, u) < 0 \end{matrix} \right\}$$

be nonempty on $\mathcal{O}_P$. If Assumption 5 holds, then the resulting closed-loop system is passive with respect to V and input–output pair (ν, y), and the safe set $\mathcal{S}$ is forward invariant despite the influence of the external disturbance.

Proof: We will first show that the set $\mathcal{S}$ is forward preinvariant for closed-loop dynamics defined by (1) and (10). Because Assumption 4 holds, Q is continuous, and $u \mapsto Q(x, u)$ is strictly convex and inf-compact for each $x \in \mathcal{O}_P$, the conditions of Lemma 1 are satisfied, and it can be concluded that the controller κ^* in (10) is single-valued and continuous on $\mathcal{O}_P$. The set-valued mapping F is outer semicontinuous by Assumptions 2 and 3, and because of the continuity of κ^*, the closed-loop dynamics defined by $F(x)$ and κ^* are continuous. By [24, Th. 1], the safe set is forward preinvariant, which means that solutions cannot escape $\mathcal{S}$ but may terminate due to finite-time escape. To conclude forward invariance, it remains to be shown that maximal solutions to the closed-loop system starting from $\mathcal{S}$ are complete. Under the continuous closed-loop dynamics defined by (1) and (10), maximal solutions are either complete or escape in finite time [36, Proposition 3]. By Assumption 5, the possibility of finite-time escape is eliminated, implying that all maximal solutions are complete. Therefore, the safe set $\mathcal{S}$ is forward invariant. Because $K_p \subseteq K$, the controller in (10) yields a controller in the set K_p. Therefore, any controller that is a selection from K ensures that the system is passive according to the definition of passivity in [9, Definition 6.3]. ∎

Remark 3: Bounded solutions avoid finite-time escape from the safe set; however, there is no guarantee that solutions to the closed-loop dynamics defined by (1) and (10) remain bounded because it is not required that $\mathcal{S}$ is designed to be bounded. There are a number of ways in which Assumption 5 can be satisfied. For example, finite-time escape is eliminated if $\mathcal{S}$ is compact or if $\mathcal{S}$ is closed and additionally the closed-loop dynamics defined by (1) and (10) are either bounded on $\mathcal{S}$ or have linear growth on $\mathcal{S}$.

With a passive control action and a state near the center of the safe set, $\kappa^* = u_{\text{nom}}$. As the state approaches the boundary of the safe set, or when the nominal control action leads to a nonpassive closed-loop system, the QP minimally modifies the nominal controller to meet both the passivity and state objectives. The implication of this control design is that the system will remain passive with respect to some external disturbance while operating within a forward invariant safe set defined by a vector-valued CBF. With the addition of the consideration of the external disturbance in the CBF design, the safe set is guaranteed to be forward invariant unlike in previous results, where there are no safeguards preventing the external disturbance from pushing the state into an unsafe region of the state space. Despite the energy being injected by the CBF, the system remains passive if there is a solution to (10).

Remark 4: With only one constraint in (10) (either the passivity, safety, or input constraint), a closed-form solution to (10) can be developed; however, developing a closed-form solution is more difficult to do with the inclusion of each constraint. Sum of squares programming can be used as in [24, Sec. V] to identify the set where at least one feasible solution to (10) exists, which is equivalent to states where K is nonempty.

Example 2: Continuing the example in Section IV-A, we will demonstrate the potential of the developed passivation approach to be used in combination with CBFs. Suppose the CBF is selected as $B(x) = [-x - \bar{x},\ x - \bar{x}]^\top$, restricting the state to $-\bar{x} \leq x \leq \bar{x}$, and γ is selected as $\gamma(x) = k_b B(x)$, where $k_b \in \mathbb{R}_{>0}$ is the CBF gain. From Section IV-A, it is known that a control input satisfying $xu \leq -x^2$ will passivate the system. Similarly, a control input of $-k_b(x + \bar{x}) + \overline{\nu} \leq u \leq -k_b(x - \bar{x}) - \overline{\nu}$ will yield forward invariance of the safe set, where $\overline{\nu} \in \mathbb{R}_{>0}$ is a bound on $\nu \in \mathbb{R}$ such that $|\nu| \leq \overline{\nu}$. It can be verified analytically that K_c is nonempty if $-k_b(x + \bar{x}) + \overline{\nu} \leq -k_b(x - \bar{x}) - \overline{\nu}$, which is true if k_b and $\bar{x}$ are selected such that $k_b \bar{x} \geq \overline{\nu}$. As can be seen in Fig. 1 , not all passivating control inputs are safe and vice versa, but a feasible solution exists at each state in the safe set.

If the $\overline{\nu}$ term was not included in the design of the CBF as in [27], the safe set of control inputs K_c would be shifted to the right, and the CBF would be unable to keep the state inside the safe set for certain values of the external disturbance. Another key advantage of the developed

[1] A function $f : X \to \mathbb{R}$ is inf-compact if for every $\lambda \in \mathbb{R}$, the sublevel set $\mathcal{L}_f(\lambda) \triangleq \{x \in X : f(x) \leq \lambda\}$ is compact.

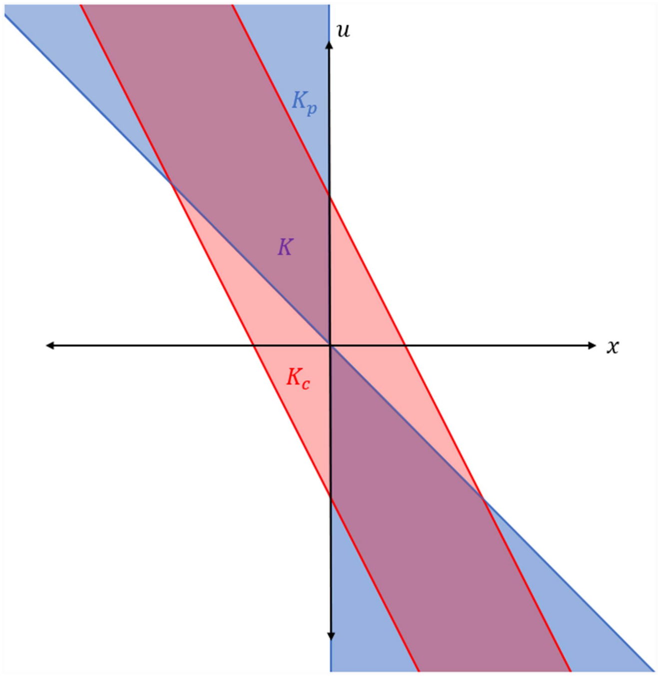

Fig. 1. Visual representation of the sets of passivating and safety-ensuring control inputs for the toy example in (9). The region outlined in blue represents K_p and the pink region between the two red lines represents K_c. The purple region represents K, where K_p and K_c overlap.

approach over previous results is its ability to pointwise minimize the control input that will achieve both objectives.

VI. Simulation

A numerical simulation was performed to provide an example of the effectiveness of the developed control scheme in ensuring both passivity of the closed-loop system as well as satisfaction of some state constraints required for safety. Consider a frictionless two-link planar and revolute robot modeled by the Euler–Lagrange dynamics in the form of [37]

$$M(q)\ddot{q} = -C(q,\dot{q})\dot{q} + \tau_e + \tau_h \tag{11}$$

where $q \triangleq [q_1\ q_2]^\top \in \mathbb{R}^2$, $\dot{q} \triangleq [\dot{q}_1\ \dot{q}_2]^\top \in \mathbb{R}^2$, and $\ddot{q} \triangleq [\ddot{q}_1\ \ddot{q}_2]^\top$ denote the angular position, velocity, and acceleration of each of the links, respectively. The inertia matrix and centripetal-Coriolis matrix are denoted by $M(q) : \mathbb{R}^2 \to \mathbb{R}^{2\times 2}$ and $C(q,\dot{q}) : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^{2\times 2}$, respectively, and are defined as

$$M(q) \triangleq \begin{bmatrix} p_1 + 2p_3\cos(q_2) & p_2 + p_3\cos(q_2) \\ p_2 + p_3\cos(q_2) & p_2 \end{bmatrix}$$

$$C(q_1,q_2) \triangleq \begin{bmatrix} -p_3\sin(q_2)\dot{q}_2 & -p_3\sin(\dot{q}_1+\dot{q}_2) \\ p_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}$$

where $p_1 = 3.473\ \mathrm{kg}\cdot\mathrm{m}^2$, $p_2 = 0.196\ \mathrm{kg}\cdot\mathrm{m}^2$, and $p_3 = 0.242\ \mathrm{kg}\cdot\mathrm{m}^2$. The electric motor torque inputs are denoted by $\tau_e \in \Psi(q) \subset \mathbb{R}^2$, where $\Psi(q) \triangleq \{\tau_e \in \mathbb{R}^2 : \|\tau_e\| \leq \overline{\tau}_e\}$ and $\overline{\tau}_e \in \mathbb{R}_{>0}$ is a user-selected upperbound on the magnitude of the control input. The inertia and centripetal-Coriolis matrix satisfy the skew-symmetric relation: $q^\top(\frac{1}{2}\dot{M}(q) - C(q,\dot{q}))q = 0$ for all $q \in \mathbb{R}^2$. The external disturbance $\tau_h \in \Phi(q) \subset \mathbb{R}^2$ can be thought of as a torque input from a human operator coming into contact with the robotic system, where $\Phi(q) \triangleq \{\tau_h \in \mathbb{R}^2 : \|\tau_h\| \leq \overline{\tau}_h\}$ and $\overline{\tau}_h \in \mathbb{R}_{>0}$ is a known upperbound on the external disturbance term. The robot needs to remain passive with respect to the human disturbance, while remaining inside some velocity bounds enforced through the use of a CBF.

The output of the system $y \in \mathbb{R}^2$ is considered to be $y = \dot{q}$, and the system's storage function is

$$V(q) = \frac{1}{2}\dot{q}^\top M(q)\dot{q}. \tag{12}$$

Invoking the skew symmetry property, the derivative of the storage function is given by $\dot{V} = \mathrm{sgn}(q)^\top\dot{q} + \dot{q}^\top(\tau_e + \tau_h)$, where $\mathrm{sgn}(\cdot)$ denotes the signum function. The storage function in (12) results in

$$P(q) = \sup_{\tau_h \in \Phi(q)} \{\dot{q}^\top\tau_h - \tau_h^\top\dot{q}\} = 0.$$

By the Definition 3, any control input in the set

$$K_p(q,\dot{q}) \triangleq \{\tau_e \in \Psi(q) : \dot{q}^\top\tau_e \leq 0\} \tag{13}$$

renders the system passive from disturbance τ_h to output $\dot{q}$.

In addition to the passivity requirement, suppose also that each link of the robot arm must comply to some energy (i.e., velocity) constraints. If procedures, such as those in [27] were followed, there would be no way to guarantee forward invariance of the safe set if there is an external disturbance from the operator; however, using the developed approach, it can be guaranteed that the state trajectories will not reach an unsafe region of the state space despite the unknown disturbance from the person. Because of physical limitations of the operator, the torque disturbance from the person τ_h can be bounded by known constants. Considering the bound of the external disturbance in the design of the CBF constraint provides robustness to the torque supplied by the operator, ensuring that the operator will not push the state outside of the safe set.

We consider a CBF candidate designed to limit the velocities of each of the links defined as $B(q,\dot{q}) \triangleq [-M(q)(\dot{q}+\bar{\dot{q}}),\ M(q)(\dot{q}-\bar{\dot{q}})]^\top$, where $\bar{\dot{q}} \in \mathbb{R}^2$ is the user-selected boundary of the safe set (i.e., the maximum allowable magnitude of the velocities of each of the links). The set of safety ensuring control inputs is defined as

$$\begin{aligned} K_c(q,\dot{q}) \triangleq \{\tau_e \in \mathbb{R}^2 : \nabla B(q,\dot{q})^\top &(-C(q,\dot{q})\dot{q} + \tau_e) \\ &\times \|\nabla B(q,\dot{q})\|\overline{\tau}_h \leq -\gamma(q,\dot{q})\} \end{aligned} \tag{14}$$

where the function γ is chosen as $\gamma(q,\dot{q}) = k_b B(q,\dot{q})$ and $k_b = 10$. The set K_c is nonempty when both components of the CBF are simultaneously feasible. The first component of the CBF, $B_1(q,\dot{q}) = -M(q)(\dot{q}+\bar{\dot{q}})$, imposes the condition on the control input $\tau_e \geq -M(q)k_b(\dot{q}+\bar{\dot{q}}) + C(q,\dot{q})\dot{q} - \dot{M}(q)(\dot{q}+\bar{\dot{q}}) + \overline{\tau}_h$. The second component of the CBF, $B_2(q,\dot{q}) = M(q)(\dot{q}-\bar{\dot{q}})$, imposes the condition on the control input $\tau_e \leq -M(q)k_b(\dot{q}-\bar{\dot{q}}) + C(q,\dot{q})\dot{q} - \dot{M}(q)(\dot{q}-\bar{\dot{q}}) - \overline{\tau}_h$. It is possible for τ_e to satisfy both inequalities only if $-M(q)k_b(\dot{q}+\bar{\dot{q}}) + C(q,\dot{q})\dot{q} - \dot{M}(q)(\dot{q}+\bar{\dot{q}}) + \overline{\tau}_h \leq -M(q)k_b(\dot{q}-\bar{\dot{q}}) + C(q,\dot{q})\dot{q} - \dot{M}(q)(\dot{q}-\bar{\dot{q}}) - \overline{\tau}_h$. Therefore, (14) is nonempty for all $q \in \mathbb{R}^2$ if k_b is selected such that $M(q)k_b\bar{\dot{q}} \geq \overline{\tau}_h$.

In this example, the QP-based control law is defined as in (10), where the cost function is defined as $Q(q,u) \triangleq \|\tau_e - u_{\mathrm{nom}}\|^2$ and the two constraints correspond to the passivity and safety conditions in (13) and (14), respectively, and the third constraint corresponds to a limit on the magnitude on the designable control input u. By defining the cost function in this way, the nominal nonpassivating controller is minimally modified such that it satisfies each constraint in the QP. We define the nominal control input as $u_{\mathrm{nom}} = [-0.1\cos(0.5t)q_1,\ 0]^\top$, which can be thought of as a variable stiffness spring acting on the first link and would not always result in passivity with respect to input–output pair $(\tau_h, \dot{q})$ and storage function (12). When $\Psi(q) = \mathbb{R}^2$, the feasibility of

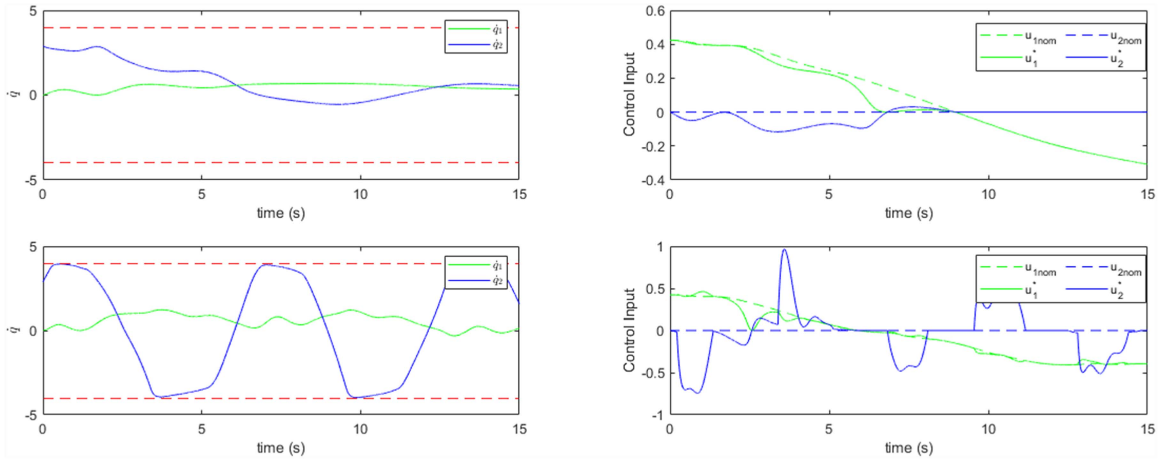

Fig. 2. Simulated evolution of the state (left) and control input (right) of the two-link manipulator system using the developed QP-based controller. The two top plots correspond to the case where human input set to zero, and the two bottom plots show the system's behavior when there is a human input.

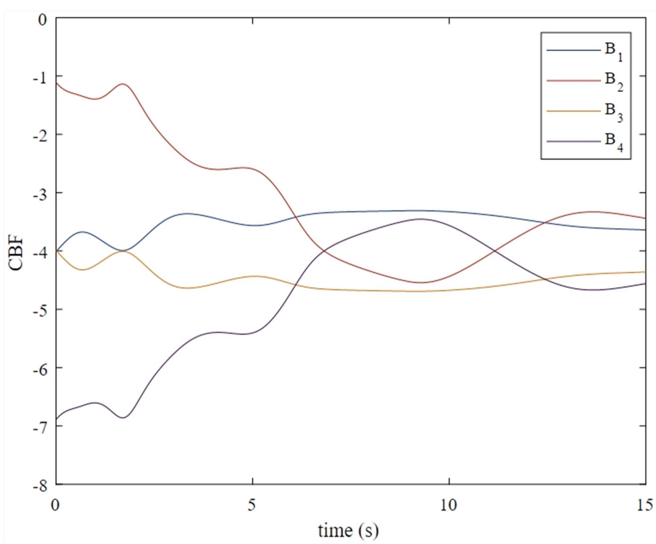

Fig. 3. Value of each of the CBFs over time. None of the CBFs reach a positive value, meaning that the state never reaches an unsafe region of the state space.

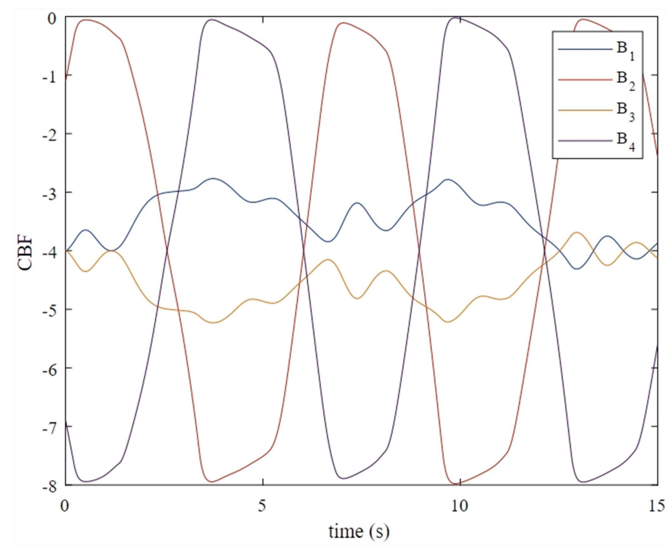

Fig. 4. Value of each of the CBFs over time with the added human input. None of the CBFs reach a positive value, meaning that the state never reaches an unsafe region of the state space.

the control law κ^* in this problem can be verified analytically. The intersection of K_p and K_c is nonempty for all $q \in \mathbb{R}^2$, provided k_b is selected such that K_c is nonempty. Because the passivity and stability constraints are both imposing conditions on $\dot{q}$ in this simulation, the feasibility analysis is simpler compared to the case of the system being rendered passive with respect to a different input–output pair. For more complex problems, the nonemptiness of K can be verified using the sum of squares programming approach developed in [24].

The two top plots of Fig. 2 show a simulation of the state and control input of the mechanical system in (11) with $\tau_h = [0 \;\; 0]^\top$, i.e., the human disturbance was set to zero. Both links started from rest, and the initial position of each of the links were randomized between $-\pi$ and π. The state trajectory stays within the desired range while remaining passive to the disturbance. The top-right plot of Fig. 2 shows how the QP modifies the control input to achieve the desired behavior. Fig. 3 shows the value of each of the CBFs over the 15-s simulation. The CBFs remain less than zero, reaching a maximum value of -1.111, meaning that the states do not leave their safe ranges.

The two bottom plots of Fig. 2 show the simulation repeated for a disturbance τ_h chosen as $\tau_h = [3\sin(3t) \;\; 4\sin(2t)]^\top$, where τ_h has an upper bound of $\overline{\tau}_h = 5$. Again, the state trajectory never exits the safe set, despite the unknown disturbance from the operator. The result is supported in Fig. 4 which shows the value of the CBF over time. For the simulated operator disturbance, the CBF reaches a maximum value of –0.023 which is closer to zero than in the case when $\tau_h = [0 \;\; 0]^\top$. This outcome is due to the use of the upper bound of the disturbance in the design of the CBF which introduces some conservativeness in the absence of disturbance.

VII. Conclusion

In this article, the concepts of PBC and CBFs are combined to produce a controller that renders the system passive with respect to an external disturbance and the safe set forward invariant. Theoretical results usually reserved for multiple CBF constraints are used to develop an optimization-based controller that enforces both passivity and state constraints. Unlike typical previous results, the external disturbance is considered during the design of the CBF to ensure the state does not reach an unsafe region of the state space at any time. The developed method was demonstrated on a two-link robotic system, yielding passivity and safety despite the unknown external disturbance by a person making contact with the robot.

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