Technical Notes and Correspondence

Adaptive Deep Neural Network-Based Control Barrier Functions

Hannah M. Sweatland, Omkar Sudhir Patil, and Warren E. Dixon

***Abstract*—Safety constraints of nonlinear control systems are commonly enforced through the use of control barrier functions (CBFs). Uncertainties in the dynamic model can disrupt forward invariance guarantees or cause the state to be restricted to an overly conservative subset of the safe set. In this article, adaptive deep neural networks (DNNs) are combined with CBFs to produce a family of controllers that ensure safety while learning the system's dynamics in real-time without the requirement for pretraining. By basing the least squares adaptation law on a state derivative estimator-based identification error, the DNN parameter estimation error is shown to be uniformly ultimately bounded. The convergent bound on the parameter estimation error is then used to formulate CBF-constraints in an optimization-based controller to guarantee safety despite model uncertainty. Furthermore, the developed method is extended for use under intermittent loss of state-feedback. A switched systems analysis for CBFs is provided with a maximum dwell-time condition during which the feedback can be unavailable. Comparative simulation results demonstrate the ability of the developed method to ensure safety when feedback is lost, unlike the baseline method. Results show improved performance compared to baseline methods and demonstrate the ability of the developed method to ensure safety in feedback-denied environments.**

***Index Terms*—Adaptive control, control barrier functions, deep neural networks.**

I. Introduction

Control barrier functions (CBFs) enforce state constraints necessary for safe operation of control systems [1], but CBF-based control input constraints depend on the dynamic model of the system. As a result, uncertainties in modeling the dynamics can endanger safety. To address challenges posed by the modeling uncertainty, robust safety methods can be used, where safety guarantees are provided using the worst-case bounds on the uncertainty. However, robust methods yield an overly conservative constraint on the control input that restricts the state to a subset of the safe set.

Adaptive CBFs have been developed to ensure the forward invariance of a safe set through online parameter adaptation [2], [3], [4]. Because the adaptive CBFs in both [2] and [3] includes the parameter estimation error, the state is restricted to a subset of the safe set, dependent on the upper-bound of the estimation error. Methods, such as set membership identification [3], integral concurrent learning [5], and parameter-adaptive CBFs [4] reduce the conservativeness with sufficient data, but these methods involve a white-box approach where the uncertainty is required to have a known structure based on traditional modeling techniques. In contrast, recent results, such as [6] and [7], use black-box models, such as pretrained deep neural networks (DNNs) and Gaussian processes, to identify uncertain dynamics using training datasets and therefore reduce conservativeness; however, since these methods are not adaptive, they result in static models that may become obsolete over time. Moreover, the offline methods often require large datasets that may not be available prior to execution in completely unknown environments.

Other recent works reduce conservativeness of the CBF-based constraints by combining CBFs with disturbance observers [7], [8], [9], [10]. Disturbance observers are used to produce an estimate of the uncertainty which is then used in the CBF constraint, expanding the state's operating region when compared to a robust approach. Adaptive safety is achieved in [7] through a modular approach that can combine a pretrained DNN model with a robust integral sign of the error (RISE)-based disturbance observer, eliminating conservativeness of the safe set over time. Although disturbance observers can estimate general nonlinear and time-varying uncertainties, the estimates are only instantaneous and do not involve a model that can be used for subsequent predictions. In contrast, models, such as DNNs, can extrapolate through unexplored regions and can be employed to ensure safety under intermittent loss of feedback. Therefore, instead of using disturbance observers or pretrained DNNs as in [8], [9], [7], and [10] it is desirable to construct adaptive CBFs using DNNs, such as those in [11], [12], [13], and [14] with analytic real-time adaptation laws without the need for pretraining. Previous Lyapunov-based (Lb-) DNN adaptive controllers address the trajectory tracking problem; however, the tracking error-based adaptation laws in such results are not suitable for the adaptive safety problem since safety does not typically require tracking error convergence. Thus, in the developed work, a novel weight adaptation law is formulated to instead yield function approximation error convergence.

In this article, adaptive DNN CBFs (aDCBFs) are developed to ensure safety while learning the system's uncertain dynamics in real-time. This article provides the first result combining CBFs with an adaptive DNN that updates in real-time, eliminating the need for pretraining. The DNN adaptation law is not based on the tracking error as in all previous Lb-DNN literature. Instead, a least squares adaptation law is designed by constructing an identification error. Since computing an identification error requires state-derivative information, an interlaced approach is used where a secondary state-derivative estimator is combined with the adaptive DNN to generate the adaptation laws. A combined Lyapunov-based analysis yields guarantees on the DNN function approximation. The convergent upper-bound of the parameter estimation error is then used to formulate candidate CBF-based constraints in an optimization-based control law to guarantee the forward invariance of the safe set, while reducing the conservative behavior often seen in robust approaches. As a result, during intermittent loss of

Received 25 November 2025; accepted 7 April 2026. Date of publication 24 April 2026; date of current version 29 September 2026. This work was supported in part by Air Force Research Laboratory (AFRL) under Grant FA8651-24-1-0018, in part by the Office of Naval Research under Grant N00014-21-1-2481, and in part by AFOSR under Grant FA9550-19-1-0169. Recommended by Associate Editor K. G. Vamvoudakis. *(Corresponding author: Hannah M. Sweatland.)*

The authors are with the Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611 USA (e-mail: hmsweatland@gmail.com; patilomkarsudhir@ufl.edu; wdixon@ufl.edu).

Digital Object Identifier 10.1109/TAC.2026.3687496

1558-2523 © 2026 IEEE. All rights reserved, including rights for text and data mining, and training of artificial intelligence and similar technologies. Personal use is permitted, but republication/redistribution requires IEEE permission. See https://www.ieee.org/publications/rights/index.html for more information.

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.

feedback, the DNN can be used to make open-loop predictions that are used to reformulate CBF-based constraints to guarantee safety. Thus, the developed method can be used for safe operation of uncertain systems in environments with feedback occlusion zones, where intermittent loss of feedback typically occurs. A switched systems analysis for CBFs is provided with a maximum dwell-time condition during which the feedback can be unavailable. Comparative simulation results are presented to demonstrate the performance of the developed method with baseline results in [7].

II. Notations and Preliminaries

Let $\mathbb{R}_{\geq 0} \triangleq [0, \infty)$, $\mathbb{R}_{>0} \triangleq (0, \infty)$, and $\mathbb{R}^{n\times m}$ represent the space of $n \times m$ dimensional matrices. The identity matrix of size n is denoted by I_n. The p-norm is denoted by $\|\cdot\|_p$, $\|\cdot\|$ is the two-norm, and $\|\cdot\|_F$ is the Frobenius norm defined as $\|\cdot\|_F \triangleq \|\text{vec}(\cdot)\|$, where $\text{vec}(\cdot)$ denotes the vectorization operator. Given some matrix $A \triangleq [a_{i,j}] \in \mathbb{R}^{n\times m}$, where $a_{i,j}$ denotes the element in the ith row and jth column of A, the vectorization operator is defined as $\text{vec}(A) \triangleq [a_{1,1}, \ldots, a_{n,1}, \ldots, a_{1,m}, \ldots, a_{n,m}]^\top \in \mathbb{R}^{nm}$. Let the notation $[d]$ be defined as $[d] \triangleq \{1, 2, \ldots, d\}$ and, for vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, let $(x, y) \triangleq [x^\top, y^\top]^\top$. For a set $B \subset \mathbb{R}^n$, the boundary of B is denoted ∂B, the interior of B is denoted $\text{int}(B)$, and an open neighborhood about B is denoted $\mathcal{N}(B)$. A set-valued mapping $M : B \rightrightarrows \mathbb{R}^m$ associates every point $x \in B$ with a set $M(x) \subset \mathbb{R}^m$.

The following control and adaptation law development can be generalized for anny network architecture Φ with corresponding Jacobian Φ'. The reader is referred to [11], [15], and [16] for full descriptions of fully connected DNNs, deep residual NNs (ResNets), and long short-term memory neural networks (NNs), respectively.

III. Problem Formulation

A. Dynamic Model

Consider the nonlinear dynamic system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state, $f : \mathbb{R}^n \to \mathbb{R}^n$ denotes an unknown continuously differentiable function, $u \in \Psi \subset \mathbb{R}^m$ denotes the control input, and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ denotes the known control effectiveness matrix, where $\Psi : \mathbb{R}^n \rightrightarrows \mathbb{R}^m$ denotes the set of admissible control inputs. The control objective is to design a controller that ensures the forward invariance of a safe set $\mathcal{S} \subset \mathbb{R}^n$ despite uncertainty in $f(x)$. Forward invariance is a common safety objective because trajectories beginning inside a forward invariant safe set will remain in the safe set.

Given a controller $\kappa : \mathbb{R}^n \to \mathbb{R}^m$ with $\kappa(x) \in \Psi(x)$ for all $x \in \mathbb{R}^n$, we refer to the closed-loop dynamics defined by (1) and κ as $f_{cl}(x) \triangleq f(x) + g(x)\kappa(x)$. A solution to the closed-loop dynamics $t \mapsto x(t)$ is complete if $\text{dom}x$ is unbounded and maximal if there is no solution y such that $x(t) = y(t)$ for all $t \in \text{dom}x$, where $\text{dom}x$ is a proper subset of $\text{dom}y$. The set $\mathcal{S}$ is forward preinvariant for the closed-loop dynamics $\dot{x} = f_{cl}(x)$ if, for each $x_0 \in \mathcal{S}$ and each maximal solution x starting from x_0, $x(t) \in \mathcal{S}$ for all $t \in \text{dom}x$ [17, Def. 2.5]. The set is forward invariant for the closed-loop dynamics if it is forward preinvariant and additionally, for each $x_0 \in \mathcal{S}$, every maximal solution x starting from x_0 is complete [17, Def. 2.6].

B. Control Barrier Functions (CBFs)

CBFs is a method used to encode a system's safety requirements. Using the development in [18], multiple scalar-valued CBF candidates can be used to define the safe set.

Definition 1 (See [18, Def. 1]): A vector-valued function $B : \mathbb{R}^n \to \mathbb{R}^d$ is a CBF candidate defining a safe set $\mathcal{S} \subset \mathbb{R}^n$ if $\mathcal{S} = \{x \in \mathbb{R}^n : B(x) \leq 0\}$, where $B(x) \triangleq (B_1(x), B_2(x), \ldots, B_d(x))$. Also let $\mathcal{S}_i \triangleq \{x \in \mathbb{R}^n : B_i(x) \leq 0\}$ and $M_i \triangleq \{x \in \partial\mathcal{S} : B_i(x) = 0\}$ for each $i \in [d]$.

The state constraints defined by the CBF candidate can then be translated to constraints on the control input, through the introduction of a performance function $\gamma : \mathbb{R}^n \to \mathbb{R}^d$. The design parameter γ limits the worst-case growth of B to ensure forward invariance of $\mathcal{S}$ based on conditions derived in [19].

Definition 2: A continuously differentiable CBF candidate $B : \mathbb{R}^n \to \mathbb{R}^d$ defining the set $\mathcal{S} \subset \mathbb{R}^n$ is a CBF for (1) and $\mathcal{S}$ on a set $\mathcal{O} \subset \mathbb{R}^n$ with respect to a function $\gamma : \mathbb{R}^n \to \mathbb{R}^d$ if 1) there exists a neighborhood of the boundary of $\mathcal{S}$ such that $\mathcal{N}(\partial\mathcal{S}) \subset \mathcal{O}$, 2) the function γ is such that, for each $i \in [d]$, $\gamma_i(x) \geq 0$ for all $x \in \mathcal{N}(M_i)\backslash\mathcal{S}_i$, and 3) the set

$$K_c(x) \triangleq \left\{u \in \Psi(x) : \nabla B^\top(x)\left(f(x) + g(x)u\right) \leq -\gamma(x)\right\} \tag{2}$$

is nonempty for every $x \in \mathcal{O}$.

Since $f(x)$ is unknown, the inequality defining K_c in (2) cannot be guaranteed to be satisfied without using a conservative bound on the dynamics that would restrict the state's operating region to a subset of the safe set. Thus, there is motivation to develop an estimate of the uncertain dynamics to expand the operating region. DNNs are a powerful tool that can be used to produce a real-time approximation of $f(x)$.

C. DNN Approximation

Based on the universal function approximation theorem, DNNs can be used to approximate continuous functions that lie on a compact set [20]. Lyapunov-based methods have been developed to update the layer weights of a number of NN architectures including fully connected DNNs [11], long short-term memory NNs [16], and ResNets [15].

On a compact set $\Omega \subset \mathbb{R}^n$, the uncertain dynamics in (1) can be modeled as

$$f(x) = \Phi\left(x, \theta^*\right) + \varepsilon(x) \tag{3}$$

where $\Phi : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ denotes the selected DNN architecture, $\theta^* \in \mathbb{R}^p$ denotes a vector of ideal weights defined as $\theta^* \triangleq \arg\min_\theta \sup_{x\in\Omega}(\|f(x) - \Phi(x, \theta)\|^2 + \sigma\|\theta\|^2)$, where $\sigma \in \mathbb{R}_{>0}$ is a regularizing constant, and $\varepsilon : \mathbb{R}^n \to \mathbb{R}^n$ denotes the unknown function reconstruction error. By the universal function approximation property, for any prescribed $\bar{\varepsilon} \in \mathbb{R}_{>0}$, there exist ideal DNN weights such that $\sup_{x\in\Omega} \|f(x) - \Phi(x, \theta^*)\| \leq \bar{\varepsilon}$. The function approximation error in (3) satisfies $\sup_{x\in\Omega} \|\varepsilon(x)\| \leq \bar{\varepsilon}$ on the compact domain Ω. The subsequent CBF analysis ensures the input to the DNN, x, remains in the forward invariant safe set $\mathcal{S} \subseteq \Omega$ for all time, so the universal function approximation property can be applied.

The DNN has a nested nonlinearly parameterized structure, so traditional adaptive control techniques used for linearly parameterized systems are not applicable. To help overcome the complexities introduced by the nonlinearities, a first-order Taylor series approximation can be used to estimate $\Phi(x, \theta^*)$ as [11]

$$\Phi\left(x, \theta^*\right) = \Phi\left(x, \hat{\theta}\right) + \Phi'\tilde{\theta} + \Delta_O^2\left(\left\|\tilde{\theta}\right\|\right) \tag{4}$$

where $\hat{\theta} \in \mathbb{R}^p$ denotes a vector composed of the adaptive estimates the DNN layer weights that are generated using the subsequently designed adaptation laws, $\Phi' \in \mathbb{R}^{n\times p}$ denotes the Jacobian of the DNN architecture defined as $\Phi' \triangleq \frac{\partial\Phi(x,\hat{\theta})}{\partial\hat{\theta}}$, $\tilde{\theta} \in \mathbb{R}^p$ denotes the weight estimation error defined as $\tilde{\theta} \triangleq \theta^* - \hat{\theta}$, and $\Delta_O^2 : \mathbb{R}^p \to \mathbb{R}^n$ denotes higher order

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.

terms. The following assumption is made to facilitate the subsequent analysis.

Assumption 1: There exists a known constant $\overline{\theta} \in \mathbb{R}_{>0}$ such that the unknown ideal weights can be bounded as $\|\theta^*\| \leq \overline{\theta}$ [21, Assumption 1]. In addition, there exists a known constant $\Xi \in \mathbb{R}_{>0}$ such that $\|f(x) - \Phi(x,\theta)\|^2 + \sigma\|\theta\|^2$ is strictly convex with respect to θ for all $\theta \in \mathcal{B} \triangleq \{\vartheta \in \mathbb{R}^p : \|\theta^* - \vartheta\| \leq \Xi\}$.

Remark 1: In Assumption 1, local convexity of the regularized loss function $\|f(x) - \Phi(x,\theta)\|^2 + \sigma\|\theta\|^2$ is assumed to facilitate convergence to a local minimum in the subsequent analysis. A number of theoretical results in deep learning literature that indicate that for some DNN architectures, such as deep ResNets, every local minimum is a global minimum [22], [23]. Furthermore, the regularizing term $\sigma\|\theta\|^2$ assists in convexifying the loss function and mitigating practical issues in deep learning, such as overfitting [24, Ch. 7]. The strict convexity assumption on the loss function ensures that θ^* is unique.

Substituting the DNN estimate in (3) and Taylor series approximation in (4) into the left-hand side of the inequality in (2) yields

$$\dot{B}(x,u) = \nabla B^\top(x)\left(\Phi\left(x,\hat{\theta}\right) + \Phi'\tilde{\theta} + \Delta + g(x)u\right) \tag{5}$$

where $\Delta \in \mathbb{R}^n$ is defined as $\Delta \triangleq \Delta_O^2(\|\tilde{\theta}\|) + \varepsilon(x)$. Although the DNN approximation alone is less conservative than bounding the entire uncertainty, (5) is still composed of the unknown terms $\tilde{\theta}$ and Δ. Assumption 1 could be used to bound $\tilde{\theta}$ in the CBF constraint in (5), but instead we introduce an adaptive identifier in the following section to further reduce conservative behavior due to the uncertainty in $\tilde{\theta}$.

D. Adaptive DNN-Based Identifier Design

All previous Lb-DNN-based adaptive control results update the DNN weights using the tracking error [11], [14], [15], [16]; however, the objective in those results is to track a desired trajectory. In many controls applications, such as adaptive safety, trajectory tracking is not an objective, so there is no notion of a tracking error. To achieve adaptive safety, the adaptive weight updates need to be performed with system identification as the objective, rather than trajectory tracking. Therefore, a least squares weight adaptation law is introduced to adaptively identify the system dynamics based on an identification error. Performing least squares-based real-time identification is challenging for continuous-time systems because it requires state-derivative information which is often unknown or noisy. Therefore, we introduce a high-gain state-derivative estimator defined as

$$\dot{\hat{x}} = \hat{f} + g(x)u + k_x\tilde{x}, \qquad \dot{\hat{f}} = k_f\left(\dot{\hat{x}} + k_x\tilde{x}\right) + \tilde{x} \tag{6}$$

where $\hat{x}, \hat{f} \in \mathbb{R}^n$ denote the observer estimates of x and f, respectively, $k_x, k_f \in \mathbb{R}_{>0}$ are positive constant observer gains, and observer errors $\tilde{x}, \tilde{f} \in \mathbb{R}^n$ are defined as $\tilde{x} \triangleq x - \hat{x}$ and $\tilde{f} \triangleq f(x) - \hat{f}$, respectively. Since state feedback is available, $\tilde{x}$ is known, but $\tilde{\hat{x}}$ is unknown. An implementable form of $\hat{f}$ can be found by integrating both sides of $\dot{\hat{f}}$ in (6) to yield $\hat{f}(t) = \hat{f}(t_0) + k_f\tilde{x}(t) - k_f\tilde{x}(t_0) + \int_{t_0}^{t}(k_fk_x + 1)\tilde{x}(\tau)d\tau$. Taking the time derivative of the definitions of $\tilde{x}$ and $\tilde{f}$ and substituting (6) yields

$$\dot{\tilde{x}} = \tilde{f} - k_x\tilde{x}, \qquad \dot{\tilde{f}} = \dot{f} - k_f\tilde{f} - \tilde{x} \tag{7}$$

where $\dot{f}(x) \triangleq \nabla f^\top(x)\dot{x}$. The following lemma is provided to establish the boundedness of $\dot{f}$ based on the continuous differentiability of f under common assumptions in CBF literature.

Lemma 1: Consider the function f, a continuous controller $\kappa \in \Psi$ and the set $\mathcal{S} \subset \mathbb{R}^n$. Based on the continuous differentiability of f, the continuity κ, and the boundedness of x on $\mathcal{S}$, the signals f and κ are bounded on $\mathcal{S}$. Therefore, there exists a known constant $\overline{\dot{f}} \in \mathbb{R}_{>0}$ such that $\|\dot{f}(x)\| \leq \overline{\dot{f}}$ for all $x \in \mathcal{S}$.

Proof: The safe set $\mathcal{S}$ is compact because it is a closed subset of the compact set Ω. Because of the continuity of f and the fact that $\mathcal{S}$ is compact, there exists a known constant $\overline{f} \in \mathbb{R}_{\geq 0}$ such that $\|f(x)\| \leq \overline{f}$ for all $x \in \mathcal{S}$. The controller κ and control effectiveness g are continuous and therefore bounded for all $x \in \mathcal{S}$. Thus, because $\dot{x} = f(x) + g(x)\kappa(x)$, it follows that $\dot{x}$ is bounded for all $x \in \mathcal{S}$ based on the boundedness of f, x, and κ on $\mathcal{S}$. The function f is continuously differentiable, so ∇f is bounded on $\mathcal{S}$. Since $\dot{f}(x) = \nabla f^\top(x)\dot{x}$, there exists a constant $\overline{\dot{f}} \in \mathbb{R}_{>0}$ such that $\|\dot{f}(x(t))\| \leq \overline{\dot{f}}$ for all $x \in \mathcal{S}$. ■

Based on the subsequent analysis, the DNN adaptation law $\dot{\hat{\theta}} \in \mathbb{R}^p$ is designed as

$$\dot{\hat{\theta}} = \text{proj}\left(\Gamma\left(-k_\theta\hat{\theta} + \alpha\Phi'^\top\left(x,\hat{\theta}\right)\left(\hat{f} - \Phi\left(x,\hat{\theta}\right)\right)\right)\right) \tag{8}$$

where $k_\theta, \alpha \in \mathbb{R}_{>0}$ denote constant gains and the projection operator $\text{proj}(\cdot)$ is defined as in [25, Appendix E] and ensures that $\hat{\theta}(t) \in \mathcal{B}$. The term $\Gamma \in \mathbb{R}^{p\times p}$ denotes a symmetric positive-definite time-varying least squares adaptation gain matrix that is a solution to

$$\frac{d}{dt}\Gamma^{-1} = -\beta(t)\Gamma^{-1} + \Phi'^\top\left(x,\hat{\theta}\right)\Phi'\left(x,\hat{\theta}\right) \tag{9}$$

with the bounded-gain time-varying forgetting factor $\beta : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ designed as $\beta(t) \triangleq \beta_0(1 - \frac{\|\Gamma\|}{\kappa_0})$, where $\beta_0, \kappa_0 \in \mathbb{R}_{>0}$ are user-defined constants that denote the maximum forgetting rate and the bound prescribed on $\|\Gamma\|$, respectively. The adaptation gain matrix is initialized to be positive-definite such that $\|\Gamma(t_0)\| < \kappa_0$, and $\Gamma(t)$ remains positive-definite for all $t \in \mathbb{R}_{\geq 0}$ [26]. Because $\Gamma(t)$ is positive-definite, there exists a constant $\kappa_1 \in \mathbb{R}_{>0}$ such that $\lambda_{\min}(\Gamma(t)) \geq \kappa_1$ for all $t \in \mathbb{R}_{\geq t_0}$. If $\Phi'(x,\hat{\theta})$ satisfies the persistence of excitation (PE) condition, meaning there exist constants $\varphi_1, \varphi_2 \in \mathbb{R}_{>0}$ such that $\varphi_1 I_p \leq \int_{t_1}^{t_1+T}\Phi'^\top(x(\tau),\hat{\theta}(\tau))\Phi'(x(\tau),\hat{\theta}(\tau))d\tau \leq \varphi_2 I_p$ for all $t_1 \in \mathbb{R}_{\geq 0}$ and $T \in \mathbb{R}_{>0}$, it can be shown that $\beta_1 > 0$ [26, Sect. 4.2], where $\beta_1 \in \mathbb{R}_{\geq 0}$ is a constant such that $\beta \geq \beta_1$.

IV. Stability Analysis

Taking the time derivative of $\tilde{\theta}$, adding and subtracting f, and substituting (3) and (4) into (8), the parameter estimation error dynamics are given as

$$\dot{\tilde{\theta}} = -\text{proj}\left(\Gamma\left(k_\theta\tilde{\theta} + \alpha\Phi'^\top\left(x,\hat{\theta}\right)\left(\Phi'\tilde{\theta} + \Delta - \tilde{f}\right) - k_\theta\theta^*\right)\right). \tag{10}$$

The subsequent Lyapunov-based stability analysis demonstrates the convergence properties of (7) and (10).

To facilitate the stability analysis, let $z \triangleq [\tilde{x}^\top\ \tilde{f}^\top\ \tilde{\theta}^\top]^\top \in \mathbb{R}^{2n+p}$ denote the concatenated state vector. Let the Lyapunov function candidate $V : \mathbb{R}^{2n+p} \to \mathbb{R}$ be defined as

$$V(z) \triangleq \frac{1}{2}\tilde{x}^\top\tilde{x} + \frac{1}{2}\tilde{f}^\top\tilde{f} + \frac{1}{2}\tilde{\theta}^\top\Gamma^{-1}\tilde{\theta} \tag{11}$$

which can be bounded as

$$\lambda_1\|z\|^2 \leq V(z) \leq \lambda_2\|z\|^2 \tag{12}$$

where $\lambda_1 \triangleq \min\{\frac{1}{2}, \frac{1}{2\kappa_0}\}$, $\lambda_2 \triangleq \max\{\frac{1}{2}, \frac{1}{2\kappa_1}\}$. Taking the time-derivative of (11), substituting (7), (9), and (10), and applying the property of projection operators $-\tilde{\theta}^\top\Gamma^{-1}\text{proj}(\mu) \leq -\tilde{\theta}^\top\Gamma^{-1}\mu$ [25, Lemma E.1.IV], yields

$$\dot{V} \leq -k_x\|\tilde{x}\|^2 - k_f\left\|\tilde{f}\right\|^2 + \tilde{f}^\top\dot{f} - \left(\frac{\beta_1}{2\kappa_0} + k_\theta\right)\left\|\tilde{\theta}\right\|^2 - \left(\alpha - \frac{1}{2}\right)\tilde{\theta}^\top\Phi'^\top\left(x,\hat{\theta}\right)\Phi'\left(x,\hat{\theta}\right)\tilde{\theta}$$

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.

$$+ \alpha \tilde{\theta}^\top \Phi'^\top \left(x, \hat{\theta}\right) \left(\tilde{f} - \Delta\right) + k_\theta \tilde{\theta}^\top \theta^*. \tag{13}$$

Because f and Φ are continuously differentiable $\|\Delta\| \leq c_1$ and $\|\Phi'(x, \hat{\theta})\|_F \leq c_2$ when $z \in \mathcal{D} \triangleq \{\zeta \in \mathbb{R}^{2n+p} : \|\zeta\| \leq \chi\}$, where $c_1, c_2, \chi \in \mathbb{R}_{>0}$ are known constants [27, Lemmas 1 and 2]. Recall that by Lemma 1, there exists a known bound $\overline{\dot{f}} \in \mathbb{R}_{>0}$ such that $\|\dot{f}\| \leq \overline{\dot{f}}$ for all $x \in \mathcal{S}$. Using Young's Inequality and Assumption 1, $\tilde{\theta}^\top \Phi'^\top(x, \hat{\theta})(\tilde{f} - \Delta) \leq c_2 \|\tilde{\theta}\|^2 + \frac{c_2}{2}\|\tilde{f}\|^2 + \frac{c_2 c_1^2}{2}$, $\tilde{f}^\top \dot{f} \leq \frac{\overline{\dot{f}}}{2}\|\tilde{f}\|^2 + \frac{\overline{\dot{f}}}{2}$, and $k_\theta \tilde{\theta}^\top \theta^* \leq \frac{k_\theta}{2}\|\tilde{\theta}\|^2 + \frac{k_\theta}{2}\overline{\theta}^2$, so (13) can be further bounded as

$$\dot{V} \leq -\lambda_3 \|z\|^2 + C - \left(\alpha - \frac{1}{2}\right) \tilde{\theta}^\top \Phi'^\top \left(x, \hat{\theta}\right) \Phi' \left(x, \hat{\theta}\right) \tilde{\theta} \tag{14}$$

where $\lambda_3 \triangleq \min\left\{k_x,\ k_f - \frac{\overline{\dot{f}}}{2} - \frac{c_2}{2},\ \frac{\beta_1}{2\kappa_0} + \frac{k_\theta}{2} - c_2\right\}$ and $C \triangleq \frac{\overline{\dot{f}} + c_2 c_1^2 + k_\theta \overline{\theta}^2}{2}$. In addition, let $\mathcal{Q} \triangleq \left\{\zeta \in \mathbb{R}^{2n+p} : \|\zeta\| \leq \sqrt{\frac{\lambda_1}{\lambda_2}\chi^2 - \frac{C}{\lambda_3}}\right\}$, which is defined to initialize z in the subsequent analysis. The following theorem provides conditions under which the adaptation law in (8) yields parameter estimation error convergence.

Theorem 1: Let $t \mapsto x(t)$ be such that $x(t_0) \in \text{int}(\mathcal{S})$[1] and there exists a time interval $\mathcal{I} \triangleq [t_0, t_\mathcal{I})$ such that $x(t) \in \mathcal{S}$ for all $t \in \mathcal{I}$. If Assumption 1 is satisfied, κ is continuous, and $\chi > \sqrt{\frac{\lambda_2 C}{\lambda_1 \lambda_3}}$, then the weight update law in (8) ensures that $\|\tilde{\theta}(t)\| \leq \tilde{\theta}_{UB}(t)$ for all $t \in \mathcal{I}$, where

$$\tilde{\theta}_{UB}(t) \triangleq \sqrt{\frac{\lambda_2}{\lambda_1}\|z(t_0)\|^2 e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2 C}{\lambda_1 \lambda_3}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right)}$$

provided $z(t_0) \in \mathcal{Q}$, $\hat{\theta}(t_0) \in \mathcal{B}$, $\lambda_3 > 0$, and $\alpha > \frac{1}{2}$.

Proof: From the Lyapunov function candidate in (11) and the inequalities in (12) and (14), $\dot{V}$ can be further bounded as $\dot{V} \leq -\frac{\lambda_3}{\lambda_2} V + C$, for all $z \in \mathcal{D}$ and $t \in \mathcal{I}$ if the gain conditions are satisfied. Solving the differential inequality over the time interval $\mathcal{I}$ yields

$$V(z(t)) \leq V(z(t_0)) e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2 C}{\lambda_3}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right) \tag{15}$$

for all $z \in \mathcal{D}$. From (11) and (15), it follows that:

$$\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}\|z(t_0)\|^2 e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2 C}{\lambda_1 \lambda_3}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right)} \tag{16}$$

for all $z \in \mathcal{D}$ and $t \in \mathcal{I}$. To ensure $z(t) \in \mathcal{D}$ for all $t \in \mathcal{I}$, further upper-bounding (16) yields $\|z(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}\|z(t_0)\|^2 + \frac{\lambda_2 C}{\lambda_1 \lambda_3}}$ for all $t \in \mathcal{I}$; hence, $z(t) \in \mathcal{D}$ always holds if $\sqrt{\frac{\lambda_2}{\lambda_1}\|z(t_0)\|^2 + \frac{\lambda_2 C}{\lambda_1 \lambda_3}} \leq \chi$, which is guaranteed if $\|z(t_0)\| \leq \sqrt{\frac{\lambda_1}{\lambda_2}\chi^2 - \frac{C}{\lambda_3}}$, i.e., $z(t_0) \in \mathcal{Q}$. Thus, trajectories of z do not escape $\mathcal{D}$ if z is initialized in $\mathcal{Q}$. In addition, because $\|\tilde{\theta}\| \leq \|z\|$, (16) implies

$$\left\|\tilde{\theta}(t)\right\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}\|z(t_0)\|^2 e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2 C}{\lambda_1 \lambda_3}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right)} \tag{17}$$

for all $t \in \mathcal{I}$ and $z \in \mathcal{D}$ if $z(t_0) \in \mathcal{Q}$. For the initial conditions to be feasible, $\mathcal{Q}$ is required to be nonempty, which is ensured by selecting $\chi > \sqrt{\frac{\lambda_2 C}{\lambda_1 \lambda_3}}$. ■

The bound in (17) cannot be implemented without information about the concatenated initial state $z(t_0)$. Since the state information is available, $\hat{x}$ is initialized such that $\tilde{x}(t_0) = 0$. While $\tilde{f}$ and $\tilde{\theta}$ are unknown, each have known bounds. Recall that because of the continuity of f and the fact that $\mathcal{S}$ is a closed subset of the compact set Ω and is therefore compact, there exists a known constant $\overline{f} \in \mathbb{R}_{\geq 0}$ such that $\|f(x)\| \leq \overline{f}$ for all $x \in \mathcal{S}$. Thus, it follows that $\tilde{f}$ is bounded. The bound on $\tilde{\theta}$ is a result of the projection operator in (10). If $\|\hat{f}(t_0)\| \leq \overline{f}$ and $\hat{\theta}(t_0) \in \mathcal{B}$, then $\|\tilde{f}(t_0)\| \leq 2\overline{f}$ and $\|\tilde{\theta}(t_0)\| \leq \Xi$. Therefore, there exists a known constant $\mathcal{Z} \in \mathbb{R}_{>0}$ such that $\|z(t_0)\| \leq \mathcal{Z} \triangleq \sqrt{\Xi^2 + 4\overline{f}^2}$ and (17) can be further bounded as

$$\left\|\tilde{\theta}(t)\right\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}\mathcal{Z}^2 e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2 C}{\lambda_1 \lambda_3}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right)} \tag{18}$$

for all $t \in \mathcal{I}$ if $z(t_0) \in \mathcal{Q}$.

Because the bound in (18) may initially be more conservative than Ξ, we design a function $\chi_\theta \in \mathbb{R}_{>0}$ as

$$\chi_\theta \triangleq \min\left\{\Xi, \sqrt{\frac{\lambda_2}{\lambda_1}\mathcal{Z}^2 e^{-\frac{\lambda_3}{\lambda_2}t} + \frac{\lambda_2 C}{\lambda_1 \lambda_3}\left(1 - e^{-\frac{\lambda_3}{\lambda_2}t}\right)}\right\} \tag{19}$$

such that $\|\tilde{\theta}(t)\| \leq \chi_\theta$ for all time. When the observer gains k_x and k_f are selected to be sufficiently high, $\lambda_3 = \frac{\beta_1}{2\kappa_0} + \frac{k_\theta}{2} - c_2$, which implies the rate of convergence in (18) depends primarily on β_1 and k_θ. Thus, when the PE condition is satisfied, $\beta_1 > 0$, resulting in a larger λ_3 which implies χ_θ converges faster and to a smaller value. When the PE condition is not satisfied, the gain k_θ helps achieve the uniform ultimate boundedness of $\tilde{\theta}$ based on sigma modification; however, selection of a larger k_θ yields a larger C, worsening parameter estimation performance. By substituting the developed upper-bound of the parameter estimation error in (19) into (5) and recalling $\|\Delta\| \leq c_1$, a new CBF notion composed of only the known signals[2] can be defined.

Definition 3: A continuously differentiable CBF candidate $B : \mathbb{R}^n \to \mathbb{R}^d$ defining the set $\mathcal{S} \subseteq \Omega$ is an *adaptive DNN CBF (aDCBF)* for the dynamics in (1) and safe set $\mathcal{S}$ on a set $\mathcal{O} \subset \mathbb{R}^n$ with respect to $\gamma : \mathbb{R}^n \to \mathbb{R}^d$ if 1) there exists a neighborhood of the boundary of $\mathcal{S}$ such that $\mathcal{N}(\partial\mathcal{S}) \subset \mathcal{O}$, 2) for each $i \in [d]$, $\gamma_i(x) \geq 0$ for all $x \in \mathcal{N}(M_i) \backslash \mathcal{S}_i$, and 3) the set

$$K_d(x) \triangleq \Big\{u \in \Psi : \left\|\nabla B^\top(x)\Phi'\right\|(\chi_\theta + c_1) + \nabla B^\top(x)\left(\Phi\left(x, \hat{\theta}\right) + g(x)u\right) \leq -\gamma(x)\Big\}$$

is nonempty for all $x \in \mathcal{O}$.

The set K_d represents the set of control inputs that will render the set $\mathcal{S}$ forward invariant. A selection of K_d that minimizes some cost function can be made at each $x \in \mathcal{O}$ using an optimization-based control law.

The controller $\kappa^* : \mathbb{R}^n \to \Psi$ is defined as

$$\kappa^*(x) \triangleq \arg\min_{u \in \Psi} Q(x, u) \quad \text{s.t. } \mathcal{C}_F(x, u) \leq 0 \tag{20}$$

where $Q(x, u) : \mathbb{R}^n \times \Psi \to \mathbb{R}$ is a cost function typically selected as $\|u - u_{\text{nom}}\|^2$, $u_{\text{nom}} \in \Psi$ is a nominal continuous control input, and $\mathcal{C}_F : \mathbb{R}^n \times \Psi \to \mathbb{R}^d$ are the constraints on the control input. By choosing $\mathcal{C}_F(x, u) \triangleq \|\nabla B^\top(x)\Phi'\|(\chi_\theta + c_1) + \nabla B^\top(x)(\Phi(x, \hat{\theta}) + g(x)u) + \gamma(x)$, the optimization problem yields a controller $\kappa^*(x) \in K_d$ for all $x \in \mathcal{O}$ assuming K_d is nonempty on the set $\mathcal{O}$. To obtain an implementable form of the controller, we impose the following conditions on the set of admissible control inputs Ψ.

[1]The initial conditions $x(t_0)$ is considered to be in the interior of $\mathcal{S}$ to ensure $\mathcal{I}$ is not measure-zero, thus ruling out solutions that instantly escape $\mathcal{S}$.

[2]If the bounds on any of the terms are unknown, an adaptive bounding technique such as the one in [28] can be used.

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.

Assumption 2: There exists a function $\psi : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^s$ such that $\Psi(x) = \{u \in \mathbb{R}^m : \psi(x,u) \leq 0\}$ for all $x \in \mathbb{R}^n$. In addition, for each $r \in [s]$, the function $u \mapsto \psi_r(x,u)$ is convex on K_d and $(x,u) \mapsto \psi(x,u)$ is continuous on $\mathcal{O} \times \mathbb{R}^m$.

The following lemma of [18, Thm. 4] provides conditions that result in a continuous κ^*.

Lemma 2 (See [18, Theorem 4]): Let $\mathcal{C} : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^h$ be continuous on $\mathcal{O} \times \mathbb{R}^m$, and, for each $g \in [h]$, let $u \mapsto \mathcal{C}_g(x,u)$ be convex on the set $K(x) \triangleq \{u \in \mathbb{R}^m : \mathcal{C}_g(x,u) \leq 0, \forall g \in [h]\}$. Suppose $Q : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ is continuous and for each $x \in \mathcal{O}$, $u \mapsto Q(x,u)$ is strictly convex and inf-compact[3] on $K(x)$. If the set $K^\circ(x) \triangleq \{u \in \mathbb{R}^m : \mathcal{C}_g(x,u) < 0, \forall g \in [h]\}$ is nonempty for every $x \in \mathcal{O}$, then $\kappa^*(x) \triangleq \arg\min_{u \in K} Q(x,u)$ is continuous.

The continuity of κ^* established in Lemma 2 allows Lemma 1 and Theorem 1 to be used in the proof of the following theorem, which presents the main result of this article. The implication of the theorem is that the developed DNN-based optimization problem in (20) ensures system trajectories beginning in the user-selected safe set $\mathcal{S}$ will remain in $\mathcal{S}$ for all time.

Theorem 2: Suppose $B : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$ is an aDCBF for the system defined by (1) and (20) defining a safe set $\mathcal{S} \subseteq \Omega$. Let $\hat{x}$, $\hat{f}$, and $\hat{\theta}$ update according to (6) and (8), respectively. Let the system be initialized such that $\hat{x}(t_0) = x(t_0)$, $\|\hat{f}(t_0)\| \leq \overline{f}$, $z(t_0) \in \mathcal{Q}$, and $\hat{\theta}(t_0) \in \mathcal{B}$. If Assumptions 1 and 2 hold, Q is selected to be continuous, and $u \mapsto Q(x,u)$ is strictly convex and inf-compact on $K_d(x)$, then the safe set $\mathcal{S}$ is forward invariant for the closed-loop dynamics defined by (1), the controller κ^* in (20), and the adaptive weight update law in (8), provided $\lambda_3 > 0$.

Proof: Let $t \mapsto x(t)$ be a solution to the closed-loop dynamics f_{cl} defined by (1) and (20). Because B is an aDCBF in the sense of Definition 3, the set $K_d(x)$ is nonempty on $\mathcal{O} \supset \mathcal{N}(\mathcal{S})$. Thus, the optimization-based control law in (20) yields a controller $\kappa^* \in K_d$. Because Q is continuous and $u \mapsto Q(x,u)$ is strictly convex and inf-compact on $K_d(x)$ by assumption, Assumption 2 holds, $\mathcal{C}_F$ is continuous on $\mathcal{O} \times \mathbb{R}^m$, and $u \mapsto \mathcal{C}_F(x,u)$ is convex on K_d. Therefore, the conditions of Lemma 2 hold, thus implying κ^* is single-valued and continuous on $\mathcal{O}$. By Lemma 1, there exists a constant $\bar{\dot{f}}$ such that $\|\dot{f}\| \leq \bar{\dot{f}}$ for all $x \in \mathcal{S}$, so Theorem 1 can be used to show that the weight update law in (8) ensures $\|\tilde{\theta}\|$ is uniformly ultimately bounded by (17) for all $t \in \mathcal{I}$. Since $\|\hat{f}(t_0)\| \leq \overline{f}$, $z(t_0) \in \mathcal{D}$, $\hat{\theta}(t_0) \in \mathcal{B}$, and $\hat{f}$ is continuously differentiable, it follows that $\|\tilde{\theta}(t_0)\| \leq \Xi$, $\|\dot{\tilde{f}}(t_0)\| \leq 2\overline{f}$, and $\|\Delta\| \leq c_1$. Therefore, for every $u \in \Psi$ and $t \in \mathcal{I}$, $\nabla B^\top(x)\dot{x} \leq \|\nabla B^\top(x(t))\Phi'\|(\chi_\theta + c_1) + \nabla B^\top(x(t))(\Phi(x(t),\hat{\theta}) + g(x(t))u)$, implying $\kappa^*(x(t)) \in K_d(x(t)) \subset K_c(x(t))$ for all $t \in \mathcal{I}$. Since K_d is nonempty for all $x \in \mathcal{O}$, K_c is nonempty on $\mathcal{O}$ and B is a CBF defining the set $\mathcal{S}$ in the sense of Definition 2. By [18, Thm. 1], $\mathcal{S}$ is forward preinvariant for the closed-loop dynamics defined by (1) and (20). Maximal solutions to f_{cl} are either complete or escape in finite-time by flowing [19, Prop. 3]. The safe set $\mathcal{S}$ is compact by definition, eliminating the possibility of finite-time escape from the safe set [29, Thm. 10.1.4], which implies all maximal solutions to the closed-loop system are complete, i.e., the maximal $\mathcal{I} = [t_0, \infty)$. Thus, it follows that the safe set $\mathcal{S}$ is forward invariant. ■

The implication of Theorems 1 and 2 is that forward invariance of the safe set $\mathcal{S}$ is guaranteed despite model uncertainty. Theorem 1 demonstrates that the developed weight adaptation law ensures the modeling error is bounded by a decreasing function, while Theorem 2 uses the bound from Theorem 1 in the CBF constraint to ensure that the system never reaches an unsafe state.

[3]A function $f : X \to \mathbb{R}$ is inf-compact if for every $\lambda \in \mathbb{R}$, the sublevel set $\mathcal{L}_f(\lambda) \triangleq \{x \in X : f(x) \leq \lambda\}$ is compact.

V. Safety Under Intermittent State Feedback

During intermittent loss of feedback, the state measurements are not available, so it is impossible to use any feedback mechanisms; however, because the developed DNN yields parameter estimation error guarantees, it can be used to make state predictions at times when feedback is lost. Let $k \in \mathbb{Z}_{\geq 0}$ denote the time index such that feedback is unavailable in the time interval $[t_{2k+1}, t_{2k+2})$. When feedback is not available, an open-loop estimation of the current state $\hat{X} \in \mathbb{R}^n$ can be updated according to $\dot{\hat{X}} = \Phi(\hat{X}, \hat{\theta}(t_{2k+1})) + g(\hat{X})u$, where the initial condition for the state estimate is $\hat{X}(t_{2k+1}) = x(t_{2k+1})$. When feedback becomes available, $\hat{X}(t)$ is reset as $\hat{X}(t) = x(t)$ for all $(t,k) \in [t_{2k}, t_{2k+1}) \times \mathbb{Z}_{\geq 0}$. In disturbance observer-based methods, such as [7], the CBF-based constraint is reliant on the observer estimate of the dynamics and therefore on state measurements, so safety cannot be ensured when feedback is lost. In the developed approach, the constraint in (20) can be modified to ensure safety until feedback is restored.

A. Modified CBF Constraint Development

The open-loop estimator error $\tilde{X} \in \mathbb{R}^n$ is defined as $\tilde{X} \triangleq x - \hat{X}$; thus, $\tilde{X}(t_{2k+1}) = 0$. Exclusively for the purpose of this section, it is assumed that there exists a known constant $\bar{u} \in \mathbb{R}^n$ such that $\|u\| \leq \bar{u}$, for all $u \in \Psi$, the drift f is globally bounded and Lipschitz, and g is globally Lipschitz. Such an assumption on the boundedness of f is mild since finite-time escape is not inherent to the uncontrolled dynamics for most physical systems of practical interest. Based the assumptions on the boundedness of u and f and the continuous differentiability of Φ, there exists a Lipschitz constant $L_U \in \mathbb{R}_{>0}$ such that

$$\left\|\Phi(x,\theta^*) - \Phi\left(\hat{X},\theta^*\right) + \left(g(x) - g\left(\hat{X}\right)\right)u\right\| \leq L_U\left\|\tilde{X}\right\| \quad (21)$$

and, additionally, there exists a constant $\Delta_U \in \mathbb{R}_{>0}$ such that

$$\left\|\Phi\left(\hat{X},\theta^*\right) - \Phi\left(\hat{X},\hat{\theta}\left(t_{2k+1}\right)\right) + \varepsilon(x)\right\| \leq \Delta_U. \quad (22)$$

Let the Lyapunov function candidate during loss of feedback $V_U : \mathbb{R}^{2n+p} \to \mathbb{R}$ be defined as $V_U \triangleq \frac{1}{2}\tilde{X}^\top\tilde{X}$. Taking the time-derivative of V_U, adding and subtracting $\Phi(\hat{X},\theta^*)$, substituting in (21) and (22), and using Young's Inequality, it can be shown that $\dot{V}_U \leq \lambda_U V_U + \frac{\Delta_U}{2}$ when feedback is unavailable, where $\lambda_U \triangleq 2L_U + \Delta_U$. Solving for V_U yields $V_U(t) \leq (V(t_{2k+1}) + \delta_U)e^{\lambda_U(t-t_{2k+1})} - \delta_U$ for all $(t,k) \in [t_{2k+1}, t_{2k+2}) \times \mathbb{Z}_{\geq 0}$, where $\delta_U \triangleq \frac{2\Delta_U}{2L_U+\Delta_U}$. Therefore, the open-loop estimation error dynamics can be shown to satisfy the form $\|\dot{\tilde{X}}\| \leq L_U\|\tilde{X}\| + \Delta_U$, which is exponentially unstable (cf., [30]) such that

$$\left\|\tilde{X}(t)\right\| \leq \overline{\tilde{X}}(t) \triangleq \sqrt{\delta_U\left(e^{\lambda_U\left(t-t_{2k+1}\right)} - 1\right)} \quad (23)$$

for all $(t,k) \in [t_{2k+1}, t_{2k+2}) \times \mathbb{Z}_{\geq 0}$. The system dynamics can be bounded as $\|\dot{x}\| \leq \|\dot{\hat{X}}\| + \|\dot{\tilde{X}}\| \leq L_U\overline{\tilde{X}} + \Delta_U + \|\Phi(\hat{X},\hat{\theta}(t_{2k+1})) + g(x)u\|$, and if ∇B is locally Lipschitz, then $\|\nabla B^\top(x) - \nabla B^\top(\hat{X})\| \leq \rho\overline{\tilde{X}}(t)$, where $\rho \in \mathbb{R}_{>0}$ is a positive constant. Thus, the controller in (20) in absences of state feedback becomes

$$\kappa^*\left(\hat{X}\right) \triangleq \arg\min_{u \in \Psi} Q\left(\hat{X},u\right) \quad \text{s.t. } \mathcal{C}_U\left(\hat{X},u\right) \leq 0 \quad (24)$$

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.

where

$$\mathcal{C}_U\left(\hat{X},u\right) \triangleq \overline{\tilde{X}}(t)\rho\Big(L_U\overline{\tilde{X}}(t)+\Delta_U+\Big\|\Phi\left(\hat{X},\hat{\theta}\left(t_{2k+1}\right)\right) + g\left(\hat{X}\right)u\Big\|\Big)+\Big\|\nabla B^\top\left(\hat{X}\right)\Big\|\left(L_U\overline{\tilde{X}}(t)+\Delta_U\right) + \nabla B^\top\left(\hat{X}\right)\left(\Phi\left(\hat{X},\hat{\theta}\left(t_{2k+1}\right)\right)+g\left(\hat{X}\right)u\right)+\gamma\left(\hat{X}\right). \quad (25)$$

The development in Sections III and IV rely on the continuity of the control input u. When feedback is lost, the controller switches, introducing discontinuities. The constraint in (25) developed for intermittent loss of feedback is empirically tested in one of the following simulations. To account for the effects of switching, the following subsection introduces a switched systems analysis for CBFs with a dwell-time condition.

B. Maximum Loss of Feedback Dwell-Time Condition

As the time without feedback increases, the worst-case bound on $\tilde{X}$ grows exponentially, causing the constraint in (25) to shrink the system's operating region. To ensure the feasibility of the controller with the modified constraint in (25), a maximum loss of feedback dwell-time condition $\Delta t_k \in \mathbb{R}_{>0}$ can be developed using (23). In this section, it is additionally assumed that B is globally Lipschitz,[4] implying that $\|\nabla B(x)\| \leq \bar{B}$, where $\bar{B} \in \mathbb{R}_{>0}$ is a known positive constant. To ensure the existence of a safety-ensuring control input, the time without feedback must be such that the inequality

$$\mathcal{C}_U\left(\hat{X},u\right) - \mathcal{C}^*(x,u) \leq \bar{\mathcal{C}} \quad (26)$$

is satisfied, where $\mathcal{C}^*(x,u) \triangleq \nabla B^\top(x)\dot{x}$ and $\bar{\mathcal{C}} \in \mathbb{R}^d$ is the user-selected maximum offset between the boundary of the safe set and the boundary of the operating region enforced by $\mathcal{C}_U$. Recall that, for the purpose of this section, the control input is assumed to be bounded such that $\|u\| \leq \bar{u}$, for all $u \in \Psi$. Substituting (25) into (26) and solving for $\Delta t_k \triangleq t_{2k+1} - t_{2k+2}$ yields a maximum loss of feedback dwell-time condition of

$$\Delta t_k \leq \frac{1}{\lambda_U}\ln\left(\frac{1}{\delta_U}\left(\left(\frac{\bar{\mathcal{C}} - 6\bar{B}\Delta_U - \mathcal{K}_U}{6L_U\bar{B}}\right)^2+1\right)\right) \quad (27)$$

where $\mathcal{K}_U \triangleq 4\bar{B}\|\Phi(\hat{X},\hat{\theta}(t_k))\| + 4\bar{B}\|g(\hat{X})\|\bar{u}$.

VI. Simulation Study

A simulation is provided to demonstrate the effectiveness of the developed aDCBFs. The optimization problem in (20) is used to define the control law with a cost function of $Q(x,u) = \|u - u_{\text{nom}}(x)\|^2$, where $u_{\text{nom}} \in \Psi$ is the nominal control input that tracks the desired trajectory.

A. Nonpolynomial Dynamics

Consider the nonlinear dynamical system in (1) with $f(x) = [x_2\sin(x_1)\tanh^2(x_2),\ x_1x_2\cos(x_2)\text{sech}(x_2)]^\top$, and $g(x) = [1,\ 1]^\top$, where $x = [x_1, x_2]^\top$. White Gaussian noise was added to the position state measurement with a signal to noise ratio of 50 dB. The position state is initialized from the uniform distribution $U(-0.2, 0.2)$ with $\dot{x}(t_0) = [0,\ 0]^\top$, and the desired trajectory is defined as $x_d(t) = 0.1t[\sin(t),\ \cos(t)]^\top$. We define a vector-valued CBF as $B(x) \triangleq [x_1 + x_2 - 2, x_1 - x_2 - 2, -x_1 + x_2 - 2, -x_1 - x_2 - 2]^\top$, which defines a diamond safe set $\mathcal{S} = \{x \in \mathbb{R}^2 : B(x) \leq 0\}$ with height and width of 4. The deep ResNet has three hidden layers, a shortcut connection across each hidden layer, and with five neurons in each layer, thus involving 174 total weights. The ResNet weights are initialized from the normal distribution $N(0, 0.5)$ and the DNN gains are selected as $\Gamma(t_0) = 5I_{174}$, $\alpha = 50$, $k_\theta = 0.001$, $\beta_0 = 2$, and $\kappa_0 = 10$. The state-derivative estimator is used to produce the secondary estimate of f using gains of $k_x = 10$ and $k_f = 5$. The nominal controller is defined as $u_{\text{nom}} = \dot{x}_d - \Phi(x,\hat{\theta}) - k_e(x - x_d)$, where $k_e = 10$. The function γ is selected as $\gamma(x) = 10B(x)$. Fig. 1 shows the safe set and the desired and actual state trajectories. The simulation was run for 20 s with a step size of 0.005 s. To simulate performance under intermittent state feedback, the state measurement is made unavailable for 1 s intervals beginning at 10 and 15 s, denoted by the orange $\times$ markers in Fig. 1. During loss of feedback, the procedure in Section V is followed, where the identified DNN is used to make a prediction of the state and the modified robust CBF constraint in (25) is used to ensure safety. The method developed in Section V prevents the state from escaping the safe set until the feedback is restored at 11 and 16 s, respectively, indicated by the purple $\circ$ markers.

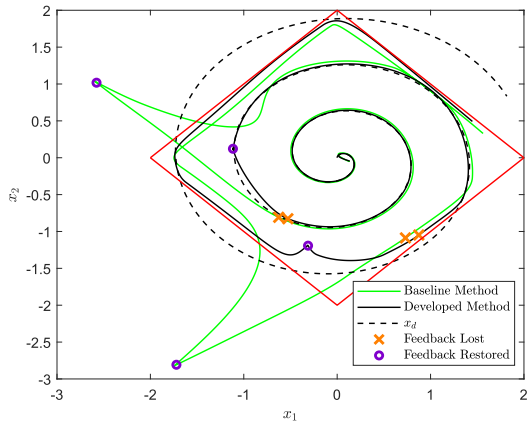


Fig. 1. State trajectory of the closed-loop system in Section VI-A using the developed aDCBF approach (black line) compared to the same control scheme without the ResNet approximation of the dynamics (green line).

For the baseline method in [7], the nominal controller is $u_{\text{nom}} \triangleq \dot{x}_d - \hat{d} - k_e(x - x_d)$, where $\hat{d} \in \mathbb{R}^2$ represents the RISE-based disturbance observer estimate of f defined in [7, eq. 6], and the CBF constraint in the optimization-based controller is $\mathcal{C}_F(x,u) \triangleq \|\nabla B^\top(x)\|\overline{f} + \nabla B^\top(x)g(x)u + \gamma(x)$. Though trajectory tracking performance is comparable to the developed approach at times when feedback is available, the state-derivative observer alone only provides an instantaneous estimate of the dynamics and fails to ensure safety in both instances of feedback loss, unlike the developed method. The basline method's violation of safety constraints is likely because the observer-based estimate of the dynamics, unlike the DNN model, does not have the capacity to generalize beyond the explored trajectory points and is therefore inaccurate under the loss of feedback. Because the developed method uses a DNN model that learns the dynamics online, the learned DNN is able to generalize on unseen trajectory points encountered during the loss of feedback. As a result, the optimization-based controllers in (20) and (24) ensure the safety constraints are satisfied with minimal modification of the nominal controller.

[4]While a global Lipschitzness requirement may appear restrictive, many safe sets of practical interest, such as polytopes (i.e., sets described by the intersection of hyperplanes) can be described using globally Lipschitz CBF candidates.

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.

VII. Conclusion

This article provides a method to ensure safety while learning the system's uncertain dynamics in real-time. The developed method is the first result that combines CBFs with an adaptive DNN that updates in real-time, eliminating the need for pretraining. Unlike previous Lb-DNN literature where the DNN adaptation laws are based on a tracking error, the developed DNN adaptation law is based on an identification error, which yields function approximation error convergence that is then used in an optimization-based control law that enforces forward invariance of the safe set. A combined Lyapunov-based analysis is proven to achieve guarantees on the DNN function approximation under the PE condition. Furthermore, to ensure safety under intermittent loss of feedback, the developed aDCBF is extended by leveraging the learned DNN's capacity to extrapolate beyond the explored trajectory points. A switched systems analysis for CBFs is provided with a maximum dwell-time condition during which the feedback can be unavailable. Simulation results are provided for an intermittent feedback examples with a comparison to observer-based CBFs as a baseline. The developed aDCBF method ensured safety even during instances of intermittent feedback, unlike the developed method.

Acknowledgment

Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the sponsoring agency.

References

[1] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Trans. Autom. Control*, vol. 62, no. 8, pp. 3861–3876, Aug. 2017.

[2] A. J. Taylor and A. D. Ames, "Adaptive safety with control barrier functions," in *Proc. Amer. Control Conf.*, 2020, pp. 1399–1405.

[3] B. T. Lopez, J.-J. E. Slotine, and J. P. How, "Robust adaptive control barrier functions: An adaptive and data-driven approach to safety," *IEEE Control Syst. Lett.*, vol. 5, no. 3, pp. 1031–1036, Jul. 2021.

[4] W. Xiao, C. A. Belta, and C. G. Cassandras, "Sufficient conditions for feasibility of optimal control problems using control barrier functions," *Automatica*, vol. 135, 2022, Art. no. 109960.

[5] A. Isaly, O. Patil, R. G. Sanfelice, and W. E. Dixon, "Adaptive safety with multiple barrier functions using integral concurrent learning," in *Proc. Amer. Control Conf.*, 2021, pp. 3719–3724.

[6] M. Srinivasan, A. Dabholkar, S. Coogan, and P. A. Vela, "Synthesis of control barrier functions using a supervised machine learning approach," in *Proc. Int. Conf. Intell. Robots Syst.*, 2020, pp. 7139–7145.

[7] A. Isaly, O. S. Patil, H. M. Sweatland, R. G. Sanfelice, and W. E. Dixon, "Adaptive safety with a RISE-based disturbance observer," *IEEE Trans. Autom. Control*, vol. 69, no. 7, pp. 4883–4890, Jul. 2024.

[8] D. R. Agrawal and D. Panagou, "Safe and robust observer-controller synthesis using control barrier functions," *IEEE Control Syst. Lett.*, vol. 7, pp. 127–132, 2023.

[9] A. Alan, T. G. Molnar, E. Das, A. D. Ames, and G. Orosz, "Disturbance observers for robust safety-critical control with control barrier functions," *IEEE Control Syst. Lett.*, vol. 7, pp. 1123–1128, 2023.

[10] J. Sun, J. Yang, and Z. Zeng, "Safety-critical control with control barrier function based on disturbance observer," *IEEE Trans. Autom. Control*, vol. 69, no. 7, pp. 4750–4756, Jul. 2024.

[11] O. Patil, D. Le, M. Greene, and W. E. Dixon, "Lyapunov-derived control and adaptive update laws for inner and outer layer weights of a deep neural network," *IEEE Control Syst. Lett.*, vol. 6, pp. 1855–1860, 2022.

[12] E. Vacchini, N. Sacchi, G. P. Incremona, and A. Ferrara, "Design of a deep neural network-based integral sliding mode control for nonlinear systems under fully unknown dynamics," *IEEE Control Syst. Lett.*, vol. 7, pp. 1789–1794, 2023.

[13] S. Li, H. T. Nguyen, and C. C. Cheah, "A theoretical framework for end-to-end learning of deep neural networks with applications to robotics," *IEEE Access*, vol. 11, pp. 21992–22006, 2023.

[14] O. S. Patil, E. J. Griffis, W. A. Makumi, and W. E. Dixon, "Composite adaptive Lyapunov-based deep neural network (Lb-DNN) controller," 2023, *arXiv:2311.13056*.

[15] O. S. Patil, D. M. Le, E. Griffis, and W. E. Dixon, "Deep residual neural network (ResNet)-based adaptive control: A Lyapunov-based approach," in *Proc. IEEE Conf. Decis. Control*, 2022, pp. 3487–3492.

[16] E. Griffis, O. Patil, Z. Bell, and W. E. Dixon, "Lyapunov-based long short-term memory (Lb-LSTM) neural network-based control," *IEEE Control Syst. Lett.*, vol. 7, pp. 2976–2981, 2023.

[17] J. Chai and R. G. Sanfelice, "On notions and sufficient conditions for forward invariance of sets for hybrid dynamical systems," in *Proc. IEEE Conf. Decis. Control*, 2015, pp. 2869–2874.

[18] A. Isaly, M. Mamaghani, R. G. Sanfelice, and W. E. Dixon, "On the feasibility and continuity of feedback controllers defined by multiple control barrier functions," *IEEE Trans. Autom. Control*, vol. 69, no. 11, pp. 7326–7339, Nov. 2024.

[19] M. Maghenem and R. G. Sanfelice, "Sufficient conditions for forward invariance and contractivity in hybrid inclusions using barrier functions," *Automatica*, vol. 124, 2021, Art. no. 109328.

[20] P. Kidger and T. Lyons, "Universal approximation with deep narrow networks," in *Proc. Conf. Learn. Theory*, 2020, pp. 2306–2327.

[21] F. L. Lewis, A. Yesildirek, and K. Liu, "Multilayer neural-net robot controller with guaranteed tracking performance," *IEEE Trans. Neural Netw.*, vol. 7, no. 2, pp. 388–399, Mar. 1996.

[22] K. Kawaguchi, "Deep learning without poor local minima," in *Proc. 30th Int. Conf. Neural Inf. Process. Syst.*, 2016, pp. 586–594.

[23] K. Kawaguchi and Y. Bengio, "Depth with nonlinearity creates no bad local minima in ResNets," *Neural Netw.*, vol. 118, pp. 167–174, 2019.

[24] I. Goodfellow, Y. Bengio, A. Courville, and Y. Bengio, *Deep Learning*, vol. 1. Cambridge, MA, USA: MIT Press, 2016.

[25] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: Wiley, 1995.

[26] J. J. Slotine and W. Li, "Composite adaptive control of robot manipulators," *Automatica*, vol. 25, no. 4, pp. 509–519, Jul. 1989.

[27] O. S. Patil, B. C. Fallin, C. F. Nino, R. G. Hart, and W. E. Dixon, "Bounds on deep neural network partial derivatives with respect to parameters," 2025, *arXiv:2503.21007*.

[28] V. Stepanyan and A. Kurdila, "Asymptotic tracking of uncertain systems with continuous control using adaptive bounding," *IEEE Trans. Neural Netw.*, vol. 20, no. 8, pp. 1320–1329, Aug. 2009.

[29] J. P. Aubin and H. Frankowska, *Set-Valued Analysis*. Cambridge, MA, USA: Birkhäuser, 2008.

[30] H.-Y. Chen, Z. Bell, P. Deptula, and W. E. Dixon, "A switched systems approach to path following with intermittent state feedback," *IEEE Trans. Robot.*, vol. 35, no. 3, pp. 725–733, Jun. 2019.

Authorized licensed use limited to: University of Florida. Downloaded on October 12,2026 at 01:17:37 UTC from IEEE Xplore. Restrictions apply.